

Theorem: $U \stackrel{d}{=} \text{Uniform}(0,1)$

F - c.d.f & F is strictly increasing

Define $X = F^{-1}(U)$

Then X has c.d.f F .

Catch:- F^{-1} may not be available explicitly

$X \stackrel{d}{=} \text{Exponential}(1)$

$$f_X(s) = \begin{cases} e^{-s} & s \geq 0 \\ 0 & \text{o.w.} \end{cases}$$

$$F(t) = \int_{-\infty}^t f_X(s) ds$$

$$= \int_0^t e^{-s} ds = 1 - e^{-t}$$

$$\begin{aligned} F^{-1}(s) &= -\ln(1-s) \\ s &\in [0,1] \end{aligned}$$

Wifi-password

ISIbccc



• Generate U

$$X = F^{-1}(u) = -\ln(1-u)$$

$$X \stackrel{d}{=} \text{Exponential}(1)$$

• Generate U

$$X = -\ln(U)$$

$$X \stackrel{d}{=} \text{Gamma}(n, 1)$$

• Generate $U_1, \dots, U_n$

$$X \stackrel{d}{=} \sum_{i=1}^n -\ln U_i = -\ln\left(\prod_{i=1}^n U_i\right)$$

$$n(1-u)$$

• Generate u

• $X = -\ln(u)$

$$n\left(\prod_{i=1}^n u_i\right)$$

Box-Mueller

Thm. - $u_1, u_2 \stackrel{d}{=} \text{Uniform}(0,1)$
independent

$$X = \sqrt{-2\ln(u_1)} \cos(2\pi u_2)$$

$$Y = \sqrt{-2\ln(u_1)} \sin(2\pi u_2)$$

X and Y are independent

& distributed as $N(0,1)$

Random variables

$$P(Y \leq t) = P(X \leq t) = \int_{-\infty}^t \frac{e^{-z^2/2}}{\sqrt{2\pi}} dz$$