

October 31st, 2019

Statistics 1

Quiz 12

<http://www.isibang.ac.in/~athreya/Teaching/statistics1>

Name: _____

Score:

1. Assume that for $\lambda > 0$, $l \in \mathbb{N}$,

$$\frac{\lambda^l}{l-1!} \int_0^1 z^{l-1} e^{-\lambda z} dz = \sum_{m=l}^{\infty} e^{-\lambda} \frac{(\lambda)^m}{m!}$$

Let $\{U_n\}_{n \geq 1}$ be Uniform $(0, 1)$ random variables. Let

$$X = \begin{cases} 0 & \text{if } \frac{-1}{\lambda} \ln(U_1) > 1 \\ \max\{j \geq 1 : \frac{-1}{\lambda} \ln(U_1 U_2 \dots U_j) \leq 1\} & \text{otherwise} \end{cases}$$

Find the distribution¹ of X

¹i.e $\text{Range}(X)$ and $\mathbb{P}(X = x)$ for all $x \in \text{Range}(X)$.