Name: _____ **Score:** ☐

1. Let X be a random variable with finite range $\{c_1, c_2\}$ for which $P(X = c_j) = p_j > 0$ for $j = 1, 2$. Let $X_1, X_2, \ldots, X_n$ be an i.i.d. sample with distribution X and let $Y_j = |\{j : X_j = c_j\}|$. Define

$$X^2 = \sum_{j=1}^{2} \frac{(Y_j - np_j)^2}{np_j}.$$

Let $F_n : [0, \infty) \to [0, 1]$ be the distribution function of X^2.

Show that there is a differentiable $F : \mathbb{R} \to [0, 1]$ such that $F_n(x) \to F(x)$ for all $x \in \mathbb{R}$ and find F'.