1. Let $R > 0$ and $X \sim$ Uniform $[0, R]$. Let $Y = \min(X, \frac{R}{10})$. Find the distribution function of Y.

Solution: Let $y < \frac{R}{10}$, then note that the event

$$\{Y \leq y\} = \{\min(X, \frac{R}{10}) \leq y\} = \{X \leq y\}.$$

So, for $y \leq \frac{R}{10}$,

$$\mathbb{P}(Y \leq y) = \mathbb{P}(X \leq y) = \begin{cases} 0 & \text{if } y \leq 0 \\ \frac{y}{R} & \text{if } 0 < y \leq \frac{R}{10} \end{cases} \tag{1}$$

Now for $y \geq \frac{R}{10}$ we have

$$\{Y \leq y\} = \{\min(X, \frac{R}{10}) \leq y\} = \{-\infty < X < \infty\}.$$

Therefore for $y \geq \frac{R}{10}$ we have that

$$\mathbb{P}(Y \leq y) = \mathbb{P}(-\infty < X < \infty) = 1. \tag{2}$$

From (1) and (2) we have

$$F_Y(y) := \mathbb{P}(Y \leq y) = \mathbb{P}(X \leq y) = \begin{cases} 0 & \text{if } y \leq 0 \\ \frac{y}{R} & \text{if } 0 < y \leq \frac{R}{10} \\ 1 & \text{if} y \geq \frac{R}{10} \end{cases} \tag{3}$$

□

Note that we have now seen an example of a random variable Y that is neither discrete nor continuous (why ?).