Ground Rules: Time allowed is 15 minutes, individual work only and closed book test. Please submit your answer as a single p.d.f. file on the Moodle platform.

1. A pair of fair dice are thrown. Let X represent the number of the first die and let Y represent the number of the second die.

- (a) Describe S, the domain of functions X and Y. How many elements are in S?
- (b) Describe T, the range of functions X and Y. How many elements are in T?
- (c) Describe the distribution of X and describe the distribution of Y by finding the probability mass function of each. Is it true that $X \sim Y$?
- (d) Are X and Y the same function? Why or why not?

Solution:

(a) Let \mathbb{N} be the set natural numbers. Then

$$S = \{(i, j) : i, j \in \mathbb{N} \text{ and } 1 \le i, j \le 6\}$$

is the sample space and the domain for both X and Y. The sample space S has 36 elements.

- (b) X and Y have the same range T where, $T = \{1, 2, 3, 4, 5, 6\}$.
- (c) For $i \in T$,

$$\mathbb{P}(X=i) = \mathbb{P}(\{(i,j) \in S : 1 \le j \le 6\}) = \frac{|\{(i,j) \in S : 1 \le j \le 6\}|}{|S|} = \frac{6}{36}.$$

For $j \in T$,

$$\mathbb{P}(Y=j) = \mathbb{P}(\{(i,j) \in S : 1 \le i \le 6\}) = \frac{|\{(i,j) \in S : 1 \le i \le 6\}|}{|S|} = \frac{6}{36}.$$

Therefore, X and Y have the same distribution. So yes $X \sim Y$.

(d) Now,

$$X: S \to T$$
, given by $X((i, j)) = i$, for all $(i, j) \in S$

and

$$Y: S \to T$$
, given by $Y((i, j)) = i$, for all $(i, j) \in S$.

Note that $X((1,2)) = 1 \neq 2 = Y((1,2))$. So they are not the same function.