**Ground Rules:** Time allowed is 15 minutes, individual work only and closed book test. Please submit your answer as a single p.d.f. file on the Moodle platform.

1. Two bookies, Gupta and Netika, bet Rupees 100 each on successive tosses of a fair coin. Each has Rupees 600 in hand and plan to keep betting until one of them runs out of money.

- 1. What is the probability that they break even after six tosses of the coin?
- 2. What is the probability that Netika wins all of the money on the tenth toss of the coin?

**Solution:** (a) Netika wins a dollar with probability  $\frac{1}{2}$  on any given toss. To break even after six tosses she must win three tosses and loose three tosses. The probability of this happening is

$$\binom{6}{3} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^3 = \frac{5}{16} = 0.3125.$$

(b) For Netika to win all of the money on the tenth toss of the coin she must win 8 tosses and loose 2 tosses. This way she ends up with  $\mathbf{\xi}_{6+\mathbf{\xi}_{8}-\mathbf{\xi}_{2}=\mathbf{\xi}_{12}}$ . She must also win the tenth toss. (If she looses toss 10 and ends up with  $\mathbf{\xi}_{12}$ , she had  $\mathbf{\xi}_{13}$  after toss 9.).

Netika must also win toss number 9. (If she looses toss 9 and wins toss 10, she had  $\overline{\mathbf{12}}$  after toss 8 and Gupta had  $\overline{\mathbf{10}}$  after toss 8.) Hence Netika must win 6 of the first eight tosses. There are  $\binom{8}{6} = 28$  different ways of doing this.

The record WWWWWWLL is forbidden, because it will imply that Gupta will run out of money on toss number 6. The other 27 ways are all possible.

The probability of any specific 10 toss outcome is  $\left(\frac{1}{2}\right)^{10}$ .

So the overall probability is

$$27\left(\frac{1}{2}\right)^{10} = \frac{27}{1024} = 0.02637$$