Due: Wednesday January 13th, 2020
Problems to be turned in Problem 2,3,4

1. Do Problem 1, Homework 4
2. Do Problem 6, Homework 4
3. Do Problem 7, Homework 4
4. The geometric distribution described the waiting time to observe a single success. A "negative binomial" distribution with parameters n and p (NegBinomial(n,p)) is defined the number of Bernoulli(p) trials needed before observing n successes. The following problem builds toward calculating some associated probabilities.
   (a) If a fair die is rolled repeatedly and a number is recorded equal to the number of rolls until the second 6 is observed, what is the sample space of possible outcomes for this experiment?
   (b) For k in the sample space you identified in part (a), what is $P(\{k\})$?
   (c) If a fair die is rolled repeatedly and a number is recorded equal to the number of rolls until the n^{th} 6 is observed, what is the sample space of possible outcomes for this experiment?
   (d) For k in the sample space you identified in part (c), what is $P(\{k\})$?
   (e) If a sequence of Bernoulli(p) trials (with $0 < p < 1$) is performed and a number is recorded equal to the number of trials until the n^{th} success is observed, what is the sample space of possible outcomes for this experiment?
   (f) For k in the sample space you identified in part (e), what is $P(\{k\})$?
   (g) Show that you have accounted for all possibilities in part (f) by showing
   $$\sum_{k\in S} P(\{k\}) = 1.$$
5. Two types of coin are produced at a factory: a fair coin and a biased one that comes up heads 55% of the time. We have one of these coins but do not know whether it is a fair or biased coin. In order to ascertain which type of coin we have, we shall perform the following statistical test. We shall toss the coin 1000 times. If the coin comes up heads 525 or more times we shall conclude that it is a biased coin. Otherwise, we shall conclude that it is fair. If the coin is actually fair, what is the probability that we shall reach a false conclusion? What would it be if the coin were biased?
6. Suppose we perform 500 independent trials with probability of success being 0.02. Using the Poisson approximation, find the probability that there are 2 or fewer successes.
7. Consider an experiment described by a Poisson$(\frac{1}{2})$ distribution and answer the following questions.
   (a) What is the probability the experiment will produce a result of 0?
   (b) What is the probability the experiment will produce a result larger than 1?