

Due: Wednesday December 30th

Problem to be turned in: 1,4

1. A box contains  $M$  balls, of which  $W$  are white. A sample of  $n$  balls, with  $n \leq W$  and  $n \leq M - W$ , is drawn at random and **without** replacement. Let  $A_j$ , where  $j = 1, 2, \dots, n$ , denote the event that the ball drawn on the  $j^{\text{th}}$  draw is white. Find  $P(A_1)$ ,  $P(A_2)$  and  $P(A_3)$ .
2. In a test called Narco-Analysis, a "truth" serum is given to a suspect. It is known that it is 90% reliable when the person is guilty and 99% reliable when the person is innocent. In other words 10% of the guilty are judged innocent by the serum and 1% of the innocent are judged guilty. If the suspect was selected from a group of suspects of which only 5% have ever committed a crime and the serum indicates that she is guilty, what is the probability that she is innocent ?
3. You first roll a fair die, then toss as many fair coins as the number that showed on the die. Given that 5 heads are obtained, what is the probability that the die showed 5 ?
4. *Polya Urn scheme*– An urn contains  $b$  black balls and  $r$  red balls. A ball is drawn at random. The ball is replaced into the urn along with  $c$  balls of its colour and  $d$  balls of the opposite colour. Then another random ball is drawn and the procedure is repeated.
  - (a) What is the probability that the second ball drawn is a black ball ?
  - (b) Assume  $c = d$ . What is the probability that the third ball drawn is a black ball ?
5. A box contains  $M$  balls, of which  $W$  are white. A sample of  $n$  balls is drawn at random, **with** replacement. Let  $A_j$ , where  $j = 1, 2, \dots, n$ , denote the event that the ball drawn on the  $j^{\text{th}}$  draw is white. Let  $B_k$  denote the event that the sample of  $n$  balls contains exactly  $k$  white balls. Find  $P(A_j|B_k)$ .
6. It is estimated that 0.8% of a large shipment of tomatoes to a certain subzi-mandi are rotten. The tomatoes are packaged in cartons, each with a dozen tomatoes, with the rotten tomatoes being randomly distributed. A retail store owner buys 10 cartons from the mandi
  - (a) If she notes the number of cartons containing rotten tomatoes, what are the possible outcomes for this experiment?
  - (b) If she notes the total number of rotten tomatoes, what are the possible outcomes for this experiment?
  - (c) How likely is it that she will find exactly one rotten tomato among all of her cartons?
  - (d) How likely is it that exactly one of her cartons will contain at least one rotten tomato?
  - (e) Explain why your answer to (d) is close to, but slightly larger than, than your answer to (c).
  - (g) What is the most likely number of rotten tomatoes she will find among her cartons?
  - (h) What is the most likely number of cartons that will contain at least one rotten tomato?
  - (i) How do you reconcile your answers to parts (g) and (h)?