Due: March 15th, 2021
Problems to be turned in Problem : None

Please solve the following questions from Class Text at: http://www.isibang.ac.in/~athreya/psweur

1. Two types of coin are produced at a factory: a fair coin and a biased one that comes up heads 55% of the time. We have one of these coins but do not know whether it is a fair or biased coin. In order to ascertain which type of coin we have, we shall perform the following statistical test. We shall toss the coin 1000 times. If the coin comes up heads 525 or more times we shall conclude that it is a biased coin. Otherwise, we shall conclude that it is fair. If the coin is actually fair, what is the probability that we shall reach a false conclusion? What would it be if the coin were biased?

2. Let Z be a standard normal random variable and $W = Z^2$.

    (a) Find $\mathbb{E}\left[e^{tW}\right]$.

    (b) Use the result of part (a) to find the variance of W.

3. Let $X \sim \text{Normal}(\mu, \sigma^2)$. Show that
$$\mathbb{P}(|X - \mu| < k\sigma)$$
   does not depend on the values of μ or σ. Can you provide bounds for $k = 1, 2, 3$?

4. Suppose $X \sim$ Poisson (λ) then for all $t \in \mathbb{R}$ find $M_X(t) = \mathbb{E}[e^{tX}]$.

5. Let $X \sim \text{Geometric}(p)$. We shall find $M_X(t)$ and use this function to calculate the expected value and variance X.

6. Let $t \in \mathbb{R}$, $X \sim \text{Normal}(0, 1)$. We shall find $M_X(t)$ and use this function to calculate the expected value and variance X. Suppose $Y = \mu X + \sigma$. Calculate $M_Y(t)$.