Probability Theory

Questions :-
Statements

(1) A bag contains 4 black balls
5 Red balls

Choose a ball at random &
the chance of drawing a red ball
is 5/9 ← $\frac{\#\text{ of favorable}}{\#\text{ of total}}$

(2) The chance of rain on Wednesday afternoon in Trivandrum is 30%. ← "relative frequency"

(3) Specificity of Rapid Antigen Test (RAT) = 0.5

Reverse transcription polymerase chain reaction (RTPCR) = 0.7

} Probability of negative test given that the patient is well

← ?

The course will make the above precise
– "Computations" might be familiar.
! – take care to pay attention to the concepts.

Probability / statistics - How likely are certain things to occur?

- one needs to know what all can occur.
- need a framework.
- Sample space, Experiment, outcome, Event, Probability.

Definition 1.1.1. (Sample Space) A sample space will be the set S is a set. The elements of S will be called outcomes & should be viewed as a listing of all possibilities that might occur. We will call the process of actually selecting one of these outcomes as an experiment.

Example:- - Roll a die and observe the outcome on the top face.

sample space - $S = \{1, 2, 3, 4, 5, 6\}$

- Toss a coin

sample space - $S = \{\text{Heads}, \text{Tails}\}$

Definition 1.1.2. [Temporary] Given a sample space S, an event is any subset $E \subseteq S$.

Example – $E = \{$ odd number shows up $\} = \{1,3,5\}$

$F = \{$ Head shows up $\} = \{$ Heads $\}$

Remarks: $S \subseteq S$ — S – event — anything can occur

$\phi \subseteq S$ — ϕ – event — "nothing occurs"

To each event we would like to assign a "likelihood" or Probability. e.g. $\mathbb{P}(\{\text{Head}\}) = 1/2$

Perform Experiment. the "chance" of heads $= 1/2$ [Relative frequency]

Consistency across events

Definition 1.1.3 [Probability Space Axioms]

Let S be a sample space.

let $\mathcal{F}$ be a collection of all events $\boxed{\mathcal{F} = \mathcal{P}(S)}$

A Probability is a function

$$\mathbb{P} : \mathcal{F} \to [0,1] \quad \text{such That}$$

(1) $\mathbb{P}(S) = 1$

(2) If $E_1, E_2, E_3 \ldots$ are a Countable Collection of disjoint events ($E_i \cap E_j = \phi$, $i \neq j$)

then
$$\mathbb{P}\left(\bigcup_{j=1}^{\infty} E_j\right) = \sum_{j=1}^{\infty} \mathbb{P}(E_j)$$

Remark:

① Axiom 1 is straight forward
S - indeed is a listing of all possible outcomes

② $\mathbb{P}\left(\bigcup_{j=1}^{\infty} E_j\right) = \sum_{j=1}^{\infty} \mathbb{P}(E_j)$

Countable union of sets

Analysis I

$x \in \bigcup_{j=1}^{\infty} E_j$

$\Rightarrow x \in E_j$ for some $j \geq 1$

Analysis 1

Series converges is absolutely.

$$\begin{cases} T_n = \sum_{j=1}^{n} \mathbb{P}(E_j) \\ T_n \to \alpha \quad \text{as } n \to \infty \end{cases}$$

$\alpha =$ " $\sum_{j=1}^{\infty} \mathbb{P}(E_j)$ "

- We will see soon that, for $k \geq 1$
$F_1, F_2, \ldots F_k$ are a finite sequence of disjoint events

$$\mathbb{P}\left(\bigcup_{j=1}^{k} F_j\right) = \sum_{j=1}^{k} \mathbb{P}(F_j)$$

(Axiom ② and ①)