1. If X_n and X all take integer values, then show that $X_n \Rightarrow X$ iff for each integer i, $P(X_n = i) \to P(X = i)$.

2. Let F be a probability distribution function on R. Define for $0 < \omega < 1$,

$$X(\omega) = \inf\{a : F(a) > \omega\}.$$

   (a) Show this is a finite real number and X is a rv on $(0, 1)$.

   (b) Show: $(0, F(c)) \quad \subset \quad \{\omega : X(\omega) \leq c\} \quad \subset \quad (0, F(c)]$

   (c) Show that X, defined on $((0, 1), \mathcal{B}, \lambda)$ has distribution function F[1].

3. Let The 'inverse' of the distribution function F be as defined above, denote Z, is called quantile function. Denote quantile functions of F_n and F by Z_n and Z respectively. If $F_n \Rightarrow F$, then show that $Z_n \to Z$ a.e. λ.

4. Suppose that $\mu_n \Rightarrow \mu$ on R and $\mathbf{H}$ is a *uniformly bounded* and *equicontinuous* family of real valued functions on R. Show that $\int f \, d\mu_n \to \int f \, d\mu$ *uniformly* in $f \in \mathbf{H}$.

[1] In other words, given any probability μ on R, there ia a random variable on $((0, 1), \mathcal{B}, \lambda)$ which has this distribution.