1. Let $X : \Omega \to \mathbb{R}$. Consider $\sigma(X)$, smallest σ-field on Ω that makes X measurable. Show

$$\sigma(X) = \{X^{-1}(B) : B \in \mathcal{B}(R)\}.$$

   Show that $Y : \Omega \to \mathbb{R}$ is $\sigma(X)$-measurable iff there is a measurable function $g : \mathbb{R} \to \mathbb{R}$ such that $Y = g \circ X$.

2. Consider $[0,1]$ interval with Lebesgue measure dx (or $d\lambda$ if you wish). Let $f : [0,1] \to \mathbb{R}$ be any continuous function. Show that if $\int_{\mathbb{R}} f(x)x^n dx = 0$ for $n = 0, 1, 2, 3, \ldots$, then $f \equiv 0$. Is the fact true if f is bounded measurable ?

3. Let $(\Omega, \mathcal{A}, P)$ be a probability space. For $n \geq 1$, let X_n be a sequence of integrable real valued random variables. Let (X_n) converges uniformly to X, i.e.

$$\sup\{|X_n(\omega) - X(\omega)| : \omega \in \Omega\} \to 0 \qquad \text{as } n \to \infty.$$

   Show that X is integrable.