You may use the programs Hermite.R, csplineclmp.R and csplineNA.R, in the shared dropbox folder to implement the cubic-spline interpolations: Hermite-cubic splines; Cubic splines Clamped End condition; and Cubic splines Natural End condition.

1. For the data below construct cubic spline with Natural End condition in the respective range. Then plot cubic polynomials along with the knots (on separate plots).

(a)

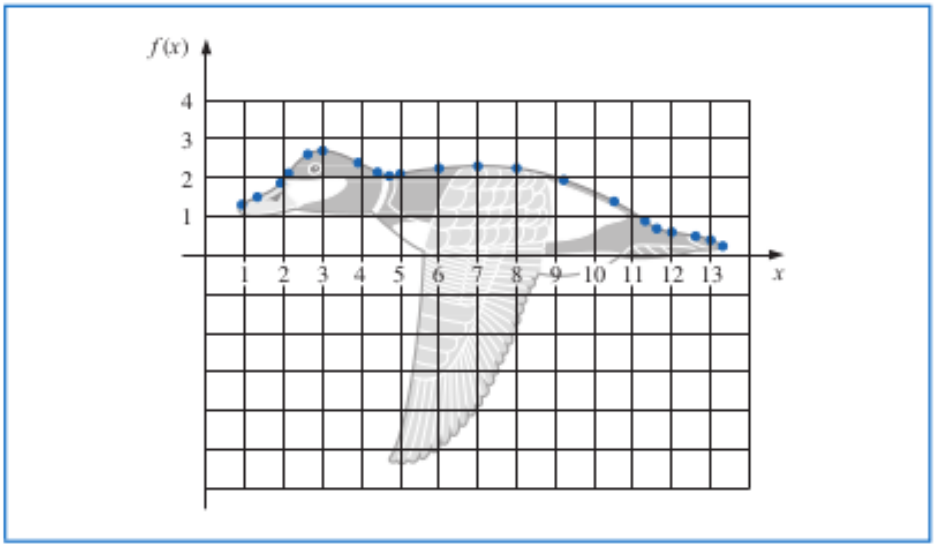
x	1	2	3
y	2	3	5

(b)

x	2.1	2.6	3.1	3.4
y	4.3	5.1	5.2	5.5

(c)
x =c(0.9,1.3,1.9,2.1,2.6,3.0,3.9,4.4,4.7,5.0,6.0,7,8,9.2,10.5,11.3,11.6,12,12.6,13,13.3)
y = c(1.3,1.5,1.85,2.1,2.6,2.7,2.4,2.15,2.05,2.1,2.25,2.3,2.25,1.95,1.4,0.9,0.7,0.6,0.5,0.4,0.25)

2. (Ruddy Duck in flight)[1]To approximate the top profile of the duck, we have chosen points along the curve through which we want the approximating curve to pass. The vectors below lists the coordinates of 21 data points relative to the superimposed coordinate system shown in Figure. Notice that more points are used when the curve is changing rapidly than when it is changing more slowly.



x=c(0.9,1.3,1.9,2.1,2.6,3.0,3.9,4.4,4.7,5.0,6.0,7.0,8.0,9.2,10.5,11.3,11.6,12.0,12.6,13.0,13.3)

y=c(1.3,1.5,1.85,2.1,2.6,2.7,2.4,2.15,2.05,2.1,2.25,2.3,2.25,1.95,1.4 0.9,0.7,0.6,0.5,0.4,0.25)

Using the above data construct and plot the cubic spline with Natural-end boundary condition. See if your profile matches the top profile of the duck.

3. Recall the function $r : [-1, 1] \to \mathbb{R}$ given by

$$r(x) = \frac{1}{1 + 25x^2}$$

given in the previous homework.

Consider equally spaced knots in the interval $[-1, 1]$ of size $4, 8, 16, 32, 64$. For each of these perform the following:

   (a) Construct the three splines.
   (b) Using the same 100 equally spaced points $\hat{x}_j$ in the interval $-1 \leq x \leq 1$, plot [in three separate plots]: the spline(s), the actual function and the error at these points between them.
   (c) Calculate 2-norm of the error in each case.

[1] image is from: Numerical Mathematics and Computing by Ward Cheney and David Kincaid.