

Timings for class

9:55 - 10:50

11:10 - 12:05

- Come on time

- Siva

AIM - Seek Solutions

to practical Problems

Numerical Methods

• Solve problems

Where theory does

not go / apply

Solution

good enough

Efficiency

Accuracy

Precision

[level of detail]

- Different from other branches of mathematics. One needs to keep track:
 - execute available theory
 - computation involved
 - answer that you find

Examples — We have seen in "theory"

[zeros] • $f: \mathbb{R} \rightarrow \mathbb{R}$ Solve for $x \in \mathbb{R}$ such
that $f(x) = 0$.

[linear system] • Solution to linear system of equations
 $A_{n \times n} x_{n \times 1} = b_{n \times 1}$ Given A & b
Find $x \in \mathbb{R}^n$

$$\{(x_i, y_i) : 1 \leq i \leq n\}$$

is there a linear relationship
between x and y ?

$$x_i \in \mathbb{R}^d, y_i \in \mathbb{R}^m$$

is there an f

such that

$$y = f(x) ?$$

- f is linear,

$$y = Ax$$

- in context
generalized least
square method

- f - polynomial
 $m=1, d=1$

- interpolation ideas
- cubic splines

- Bases in
function
space

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Riemann
Integral

$$\int_a^b f(x) dx$$

$f: \mathbb{R} \rightarrow \mathbb{R}$ - bounded and continuous.
 $a, b \in \mathbb{R}$

- Symbolic computation
 - Riemann sum
- } - Find the answer.

Differential
Equations

$$\begin{cases} y: [0, T] \rightarrow \mathbb{R} \\ \frac{dy}{dt} = f(y(t)) \end{cases}$$

$f: \mathbb{R} \rightarrow \mathbb{R}$ to be bounded
(continuous)

$$y(t) = y_0 + \int_0^t f(y(s)) ds$$

For each of the above examples:

— Explore the theory: where it begins/ends

Computation is not
manually feasible.

Find "the" answer

— correct: — then get as close to
it as possible

Question: What are the limitations?

Machines - can only store numbers that
 can be represented by finitely many
 digits

[max number of bits] $\mathbb{R}$

floating point
Real line

largest number

smallest number

$\mathbb{R}$ $a, b \in \mathbb{R}$

$\# \{x \in \mathbb{R} \mid a \leq x \leq b\}$ is finite
 could be $\geq 2^{\infty}$

$\mathbb{R}$ -line

$a, b \in \mathbb{R} \quad a < b$

$\# \{x \in \mathbb{R} \mid a \leq x \leq b\}$
 is infinite

Understanding various
 infinities

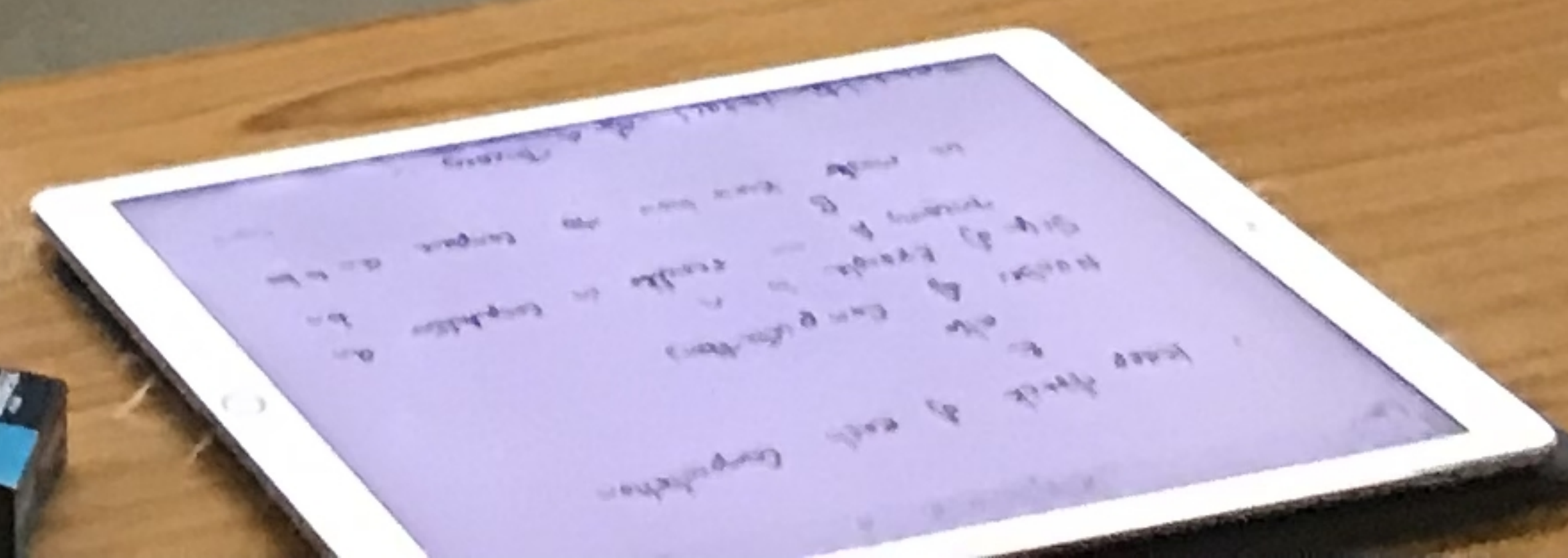
- Range is limited in fIR
- values computed using standard arithmetic $+, -, \times, \div$ in fIR have issues

$(\mathbb{R}, +, \times, -, \div)$

$\downarrow$
 fIR is not necessarily closed under arithmetic operations

fIR — Error introduces — Round off Error — { Cancellation catastrophe
 no closure wrt operation

Trunc
 NEED:
 keep
 Size
 Theory
 Theory



Truncation Error } ^{Eg} f - Taylor series

$P_n(\cdot)$ - Taylor polynomial

$R_n(\cdot)$ - Remainder [Error]

NEED:-

• Keep track of each computation

& # of computations.

• Size in each example - n

keep track of how to
compare a_n vs b_n

Theory A — requires a_n number of computations
Theory B — requires b_n number of computations

Stability

$A_{n \times n}$

$b_{n \times 1}$

Find $x_{n \times 1}$

$$Ax = b$$

- we know x exists & Computational procedure is available.

$$A = [a_{ij}] \rightarrow \tilde{a}_{ij} = a_{ij} \pm \frac{1}{10^{10}} \rightarrow \tilde{A} = [\tilde{a}_{ij}]$$

b

$$\tilde{b} = b + \epsilon$$

$$\tilde{A}\tilde{x} = \tilde{b}$$

$\tilde{x}$ also exists

Is x close to $\tilde{x}$?