Computer Science II **February 20th, 2020** **Quiz 5** **Score**

Name: Solution

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1. Solve the

$$\int_0^1 (x^2+2)^{-1} dx$$

via the following methods:

(a) Calculate the integral using Simpson rule.

(b) Give a precise formula for the error in approximation.

Answer: Let $f : [0,1] \to \mathbb{R}$ be given by $f(x) = (x^2+2)^{-1}$. Then by Simpson rule

$$\int_0^1 (x^2+2)^{-1} dx = \frac{1}{6}(f(0) + 4f(0.5) + f(1)) - \frac{1}{(2^5)(90)} f^{(4)}(\xi),$$

with $\xi \in (0,1)$. Now

$$f(0) = ((0)^2+2)^{-1} = \frac{1}{2}, \qquad f(0.5) = ((0.5)^2+2)^{-1} = \frac{4}{9}, \qquad \text{and } f(1) = ((1)^2+2)^{-1} = \frac{1}{3}.$$

Further

$$f^{(1)}(x) = \frac{-2x}{(x^2+2)^2}, \qquad f^{(2)}(x) = \frac{-2}{(x^2+2)^2} + \frac{8x^2}{(x^2+2)^3}, \qquad f^{(3)}(x) = \frac{8x}{(x^2+2)^3} + \frac{16x}{(x^2+2)^3} - \frac{48x^3}{(x^2+2)^4}$$

$$f^{(4)}(x) = \frac{24}{(x^2+2)^3} - \frac{144x^2}{(x^2+2)^4} - \frac{144x^2}{(x^2+2)^4} + \frac{384x^4}{(x^2+2)^5}.$$

(a) $S^{(0,1)}(f) = \frac{1}{6}(\frac{1}{2} + \frac{16}{9} + \frac{1}{3})$.

(b) Error $= -\left[\frac{1}{(2^5)(90)}\left(\frac{24}{(\xi^2+2)^3} - \frac{288\xi^2}{(\xi^2+2)^4} + \frac{384\xi^4}{(\xi^2+2)^5}\right)\right]$.