

Name: Solution

1. We wish to approximate the value of c which is the point on the graph of $y = x^2$ that is closest to (1,0). Fill in the blanks.

(a) If we were to represent the problem as finding the zero of $f:[a,b]\to \mathbb{R}$ then

$$a = \boxed{0}, b = \boxed{1}, \qquad f(x) = \boxed{2x^3 + x - 1}$$

(b) If x_n represents the *n*-th iterate of the Newton-Raphson method for the finding the zero then

$$x_{n+1} = \boxed{x_n - \frac{2x_n^3 + x_n - 1}{6x_n^2 + 1}}$$

(c) If y_n represents the *n*-th iterate of the Secant method for the finding the zero then

$$y_{n+1} = y_n - \frac{(2y_n^3 + y_n - 1)(y_n - y_{n-1})}{2y_n^3 + y_n - 1 - (2y_{n-1}^3 + y_{n-1} - 1)}$$

and

$$|y_{n+1} - z| \le M \left(|y_n - \boxed{z}|\right)^{\underbrace{1+\sqrt{5}}{2}}$$

for suitable M > 0, and $z \in \mathbb{R}$ is the *x*-coordinate of the point on the curve closest to (0, 1)