Due Date: March 19th, 2020
Problems due: 2,4,6

1. Complete the in-Class Worksheet
2. For each of the functions below find the coefficients of the cubic splines with Natural-End and Clamped end condition.
   (a) $f : [0, 8] \to \mathbb{R}$ given by $f(x) = \sqrt{x+1}$ with knots {0,3,8}.
   (b) $f : [0, 0.9] \to \mathbb{R}$ given by $f(x) = \cos(x^2)$ with knots {0,0.6,0.9}.
3. Construct the cubic spline with Natural-End condition for the following data:
   (a) $f(0) = 1,\ f(0.5) = 271828$
   (b) $f(-0.25) = 1.44,\ f(0.25) = 28$
4. Does there exist a, b, c, d such that the function
$$S(x) = \begin{cases} ax^3 + x^2 + cx & \text{if } -1 \leq x \leq 0 \\ bx^3 + x^2 + dx & \text{if } 0 \leq x \leq 1 \end{cases}$$
   is the cubic spline with Natural-End condition that agreess with $|\ x\ |$ at the knots $-1, 0, 1$?
5. Determine the varules of a, b, c, and d such that S is a cubic cpline and such that
$$\int_0^2 [S''(x)]^2 dx$$
   is a minimum:
$$S(x) = \begin{cases} 3 + x - 9x^3 & \text{if } 0 \leq x \leq 1 \\ a + b(x-1) + c(x-1)^2 + d(x-1)^3 & \text{if } 1 \leq x \leq 2 \end{cases}$$
6. List the ways in which the following functions fail to be Cubic splines with natural end condition:
$$S(x) = \begin{cases} x + 1 & \text{if } -2 \leq x \leq -1 \\ x^3 - 2x + 1 & \text{if } -1 \leq x \leq 1 \\ x - 1 & \text{if } 1 \leq x \leq 2 \end{cases} \qquad S(x) = \begin{cases} x^3 + x - 1 & \text{if } -1 \leq x \leq 0 \\ x^3 - x - 1 & \text{if } 0 \leq x \leq 1 \end{cases}$$
7. A strictly diagonally dominant matrix $A_{n\times n} = [a_{ij}]_{1\leq i,j\leq n}$ is one in which
$$|\ a_{ii}\ | > \sum_{j=1, j\neq i}^{n} |\ a_{ij}\ |$$
   for all $1 \leq i \leq n$. Show that every strictly diagonally dominant matrix is non-singular.
8. Consider $f : [1, 2] \to \mathbb{R}$ given by
$$f(x) = \frac{1}{x},$$
   for $x \in [1, 2]$. Let P be the polynomial of degree less than or equal to n that interpolates the points $\{(x_i, f(x_i) : 0 \leq i \leq n\}$. Let $e : [1, 2] \to \mathbb{R}$ be given by
$$e(x) = f(x) - p(x).$$
   for $x \in [1, 2]$. Show that there exists a choice of $x_0, x_1, \ldots, x_n$ such that $|\ e(x)\ | < \frac{1}{2^n}$ for all $x \in [1, 2]$?