Computer Science II | Numerical Methods | Semester II 2019/20

http://www.isibang.ac.in/~athreya/Teaching/cs219 | Homework

Due Date: March 12th, 2020

Problems due: 2,3

1. Complete the in-Class Worksheet
2. Write a Rcode called `wiggle`, with input parameter n, to perform the following tasks.
   (a) Compute n equally spaced points x_k values ($k = 1, \ldots, n$) on the interval $-1 \leq x \leq 1$.
   (b) Evaluate $r(x_k)$ where $r : [-1, 1] \to \mathbb{R}$ given by $r(x) = \frac{1}{1+25x^2}$.
   (c) Use the n pairs of $(x_k, r(x_k))$ values to define a $n - 1$ degree polynomial interpolant, P_{n-1}.
   (d) Create 100 equally spaced points $\hat{x}_j$ values ($j = 1, \ldots, 100$) in the interval $-1 \leq x \leq 1$ and evaluate $P_{n-1}(\hat{x}_k)$.
   (e) Plot $(x_k, r(x_k)), 1 \leq k \leq 10$ with open circles; $(\hat{x}_j, r(\hat{x}_j)), j = 1, \ldots, 100$ with solid line; and $(\hat{x}_j, P_{n-1}(\hat{x}_j)), j = 1, \ldots, 100$ with dashed line.
   (f) Print the value of $\| r(\hat{x}) - P_{n-1}(\hat{x}) \|_2$.

   Run your `wiggle` function and see if you spot a wiggle effect for n = seq(5,15, by =2) and see the behaviour of (f).
3. For a function f, the divided differences are given by

$$\begin{array}{llll} x_0 = 0 & f[x_0] & & \\ & & f[x_0, x_1] & \\ x_1 = 0.4 & f[x_1] & & \frac{50}{7} \\ & & 10 & \\ x_2 = 0.7 & 6 & & \end{array}$$

   Find the missing values.
4. Show that the polynomial interpolating the following data has degree 3.

x	-2	-1	0	1	2	3
f(x)	1	4	11	16	13	-4

5. For data in Table (a) compute manually the (interpolating) Newton polynomial. For data in Table (b) use the program written in class-worksheet to construct the (interpolating) Newton polynomial.

(a)

x	f(x)
-0.1	5.3
0	2
0.2	3.19
0.3	1

and

(b)

x	f(x)
0	-6
0.1	-5.89483
0.3	-5.65014
0.6	-4.28172

6. Recall the error bound derived for the Lagrange interpolating polynomial. Let $f : [a, b] \to \mathbb{R}$, $x, x_i \in [a, b]$ $i = 0, \ldots, n$. Show that there is a there is a $\xi \in [a, b]$ such that

$$f[x_0, x_1, x_2, x_3, \ldots, x_n, x] = \frac{f^{(n+1)}(\xi)}{(n+1)!}$$

7. Find the Cubic Spline $S : [1, 3] \to \mathbb{R}$ for the points

$$(1, 2), (2, 3), (3, 5)$$

   with the *Clamped*-end point condition. Namely

$$s'(1) = 2 \qquad \text{and} \qquad s'(3) = 1$$