Due Date: February 6th, 2020.
Problems due: 3,4

1. Finish the inclass worksheet.
2. Write a function in R called secant.R and implement the Secant method and find the zero when convergence criteria is decided by the iterate difference below tol =1e-5 or the maximum number of iterations exceeds 50. Use it to find the non-zero root of the function $f : [-\pi, \pi] \to \mathbb{R}$ given by

$$f(x) = x^3 - (\sin(x))^2.$$

   Compare your answer with the Bisection method and Newton-Raphson method done in the worksheet.
3. For each of the function $f : [-2, 2] \to \mathbb{R}$ given by

$$(i) f(x) = x^3 - x \qquad (ii) f(x) = x^2 - x.$$

   The functions have multiple roots.

   (a) Implement the Bisection method via the code bisect.R as in the worksheet and see if there is a preferred root chosen by the method.

   (b) Implement Newton-Raphson method via the code newton.R as in the worksheet for both functions with various starting points and see if there is convergence always and if so is there a preferred root.
4. Let $f : \mathbb{R} \to \mathbb{R}$ and f be twice differentiable with continuous second derivative. Suppose
   - there is a $c \in \mathbb{R}$ such that $f(c) = 0$.
   - there is a $\delta > 0$, such that $f'(x) \neq 0$ for all $x \in (c - \delta, c + \delta)$

   Show that there is an $\eta > 0$ such that if $x_0 \in (c - \eta, c + \eta)$ then

   (a) for all $k \geq 1$,

$$x_k = x_{k-1} - \frac{f(x_{k-1})}{f'(x_{k-1})}$$

   is a well defined sequence of real numbers;

   (b) there is a $M > 0$ such that

$$| x_k - c | \leq M(x_{k-1} - c)^2$$

   for all $k \geq 1$; and

   (c) $x_k \to c$ as $k \to \infty$.
5. Let $f : \mathbb{R} \to \mathbb{R}$ and f be twice differentiable with continuous second derivative. Suppose
   - there is a $c \in \mathbb{R}$ such that $f(c) = 0$.
   - there is a $\delta > 0$, such that $f'(x) \neq 0$ for all $x \in (c - \delta, c + \delta)$

   Show that there is an $\eta > 0$ such that if $x_0 \in (c - \eta, c + \eta)$ then

   (a) for all $k \geq 1$,

$$x_k = x_{k-1} - \frac{f(x_{k-1})}{f[x_{k-1}, x_{k-2}]}$$

   is a well defined sequence of real numbers;

   (b) there is a $M > 0$ such that

$$| x_k - c | \leq M \mid x_{k-1} - c \mid^p$$

   for all $k \geq 1$ with $p = \frac{1+\sqrt{5}}{2}$; and

   (c) $x_k \to c$ as $k \to \infty$.