Due Date: January 30th, 2020.
Problems due: 2, 3(a), 3(g)

1. Finish the inclass worksheet.
2. Suppose that $n \geq 2$, $f : \mathbb{R} \to \mathbb{R}$ and $a \in \mathbb{R}$ is such that, $f^{(k}(a) = 0$ for all $k \leq n-1$ and $f^{(n)}(a) \neq 0$. If $f^{(n)}(\cdot)$ is continuous at a then show that
   (a) if n is even and $f^{(n)}(a) > 0$ then f has a local minimum at a,
   (b) if n is even and $f^{(n)}(a) < 0$ then f has a local maximum at a,
   (c) if n is odd then f has a point of inflection at a.
3. Let $f, g : \mathbb{N} \to \mathbb{R}_+$. We define
   - $f(n) = O(g(n))$ if there exists $N_0 \in \mathbb{N}$ and $c > 0$ such that $f(n) \leq cg(n)$ for all $n \geq N_0$
   - $f(n) = \Omega(g(n))$ if there exists $N_0 \in \mathbb{N}$ and $c > 0$ such that $f(n) \geq cg(n)$ for all $n \geq N_0$
   - $f(n) = \Theta(g(n))$ if there exists $N_0 \in \mathbb{N}$ and $c_1, c_2 > 0$ such that $c_1 g(n) \leq f(n) \leq c_2 g(n)$ for all $n \geq N_0$
   - $f(n) = o(g(n))$ if for every $c > 0$ there exists N_0 such that $f(n) \leq cg(n)$ for all $n \geq N_0$

   For each of the following indicate whether $f(n) = O(g(n))$, $f(n) = \Omega(g(n))$,$f(n) = \Theta(g(n))$,$f(n) = o(g(n))$
   (a) $f(n) = 100n$ and $g(n) = n^{1.1}$
   (b) $f(n) = nb^n$, for $b \in (0, 1)$ and $g(n) = \frac{1}{n^3}$
   (c) $f(n) = 2^n$ and $g(n) = \sqrt{n}^{\sqrt{n}}$
   (d) $f(n) = n^{\log_2 \log_2 n}$ and $g(n) = 2(\log_2 n)^{\log_2 n}$
   (e) $f(n) = n$ and $g(n) = \frac{2n!}{n^{2n}}$
   (f) $f(n) = n^3 + 2n^2 + 10$ and $g(n) = (\log_2 n)^5$
   (g) $f(n) = \frac{2^n}{n!}$ and $g(n) = n^{-0.9n}$
4. *Extra Credit*[2] The Euler-Mascheroni constant is defined as

$$\gamma := \lim_{n\to\infty} \gamma_n$$

   with

$$\gamma_n = \sum_{k=1}^{n} \frac{1}{k} - \ln(n).$$

   (a) Prove that γ_n converges to a $\gamma \in \mathbb{R}$ as $n \to \infty$ and that $\gamma \geq \frac{1}{2}$.
   (b) Write a R-code for computing the constant γ using the criteria for convergence defined by the n at which $\gamma_n - \gamma_{n-1}$ is less than a pre-specified tolerance. Try to ensure minimal round-off error while executing the above.
   (c) Further decide if at your tolerance-value whether the convergence criteria yields a good estimate. The constant up to 50 Decimal places is known to be

$$\gamma = 0.57721566490153286060651209008240243104215933593992\ldots$$

   (d) Can you adjust your code to find the best possible match in the floating point number line to the Euler constant ?
   (e) Any thoughts on whether: γ is rational or irrational or algebraic or transcendental ? If you have a definite answer then you can add to the survey[3] https://arxiv.org/pdf/1303.1856.pdf which contains other interesting known facts and conjectures about the constant. For all else there is the entry in Wikipedia at https://en.wikipedia.org/wiki/Euler%E2%80%93Mascheroni_constant. Happy reading!.

[2]These are not part of the credit towards this Homework. You may attempt any of them as you feel appropriate and you can discuss solutions with me before writing it up. Selected solutions will be posted on the course website. If attempted with enthusiasm and solutions given precisely then I will bake a chocolate cake at the end of the semester for the class.
[3]Published at: https://www.ams.org/journals/bull/2013-50-04/S0273-0979-2013-01423-X/S0273-0979-2013-01423-X.pdf