

1. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a Probability space. Suppose X, Y are independent random variables on $(\Omega, \mathcal{F}, \mathbb{P})$ and $f : \mathbb{R} \rightarrow \mathbb{R}$ be a Borel-measurable function. Show $f(X)$ and $f(Y)$ are also independent.
2. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a Probability space and $\{Y_n\}$ be a sequence of random variables on $(\Omega, \mathcal{F}, \mathbb{P})$.
 - (a) Show that $E = \{\omega \in \Omega : \exists Y(\omega) \text{ s.t. } Y_n(\omega) \rightarrow Y(\omega) \text{ as } n \rightarrow \infty\}$ is a measurable set.
 - (b) Show $\bar{Y} := \limsup_{n \rightarrow \infty} Y_n$ is a random variable.
 - (c) Show $\underline{Y} := \liminf_{n \rightarrow \infty} Y_n$ is a random variable.

1.