C-12 Probability Theory **September 1, 2022** **Quiz 5** **Score**

Name ______________________ **Seat**

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1. Suppose we are given a sequence of random variables $\{Z_n\}, Z$ such that $E(\min(|Z_n - Z|, 1), \to 0$. Then show that $\{Z_n\}$converges in probability to Z.

Solution: let $0 < \varepsilon < 1$

$$E[\min(|Z_n - Z|, 1)] = E[\min(|Z_n - Z|, 1) \mathbb{1}_{\min(|Z_n - Z|, 1) < \varepsilon}] + E[\min(|Z_n - Z|, 1) \mathbb{1}_{\min(|Z_n - Z|, 1) \geq \varepsilon}]$$

$$\geq 0 + \varepsilon \, \mathbb{P}(\min(|Z_n - Z|, 1) \geq \varepsilon)$$

As $|Z_n - Z| \leq \min(|Z_n - Z|, 1)$

$$\Rightarrow \quad E[\min(|Z_n - Z|, 1)] \geq \varepsilon \, \mathbb{P}(|Z_n - Z| \geq \varepsilon)$$

$$\therefore \quad 0 \leq \mathbb{P}(|Z_n - Z| \geq \varepsilon) \leq \frac{E[\min(|Z_n - Z|, 1)]}{\varepsilon} \quad \text{—} (1)$$

$\forall n \geq 1$

$\forall 0 < \varepsilon < 1$

$\varepsilon \geq 1$

$$0 \leq \mathbb{P}(|Z_n - Z| \geq \varepsilon) \leq \mathbb{P}(|Z_n - Z| \geq \tfrac{1}{2}) \overset{(1)}{\leq} 2 E[\min(|Z_n - Z|, 1)] \quad \text{—} (2)$$

As $E[\min(|Z_n - Z|, 1)] \to 0$ as $n \to \infty$

from (1) and (2) $\mathbb{P}(|Z_n - Z| \geq \varepsilon) \to 0$ as $n \to \infty$ $\forall \varepsilon > 0$

$$\Rightarrow \quad Z_n \xrightarrow{P} Z \text{ as } n \to \infty$$