1. Suppose μ is a measure defined on $(\mathbb{N}, \mathcal{P}(\mathbb{N}))$. Suppose $A_n \in \mathcal{P}(\mathbb{N}), A_1 \supseteq A_2 \supseteq \ldots$ and $A = \cap_{n=1}^{\infty} A_n$ then is it true that $\mu(A) = \lim_{n\to\infty} \mu(A_n)$?

Solution: Let $\mu : \mathcal{P}(\mathbb{N}) \longrightarrow [0,\infty) \cup \{\infty\}$ be given by

$\mu(A) = |A|$ (i.e. Counting measure)

Ex: Verify μ is a measure

$A_n = \{k \in \mathbb{N} \mid k \geq n\}$

$A_n \supseteq A_{n+1}$ $\forall n \geq 1$ and $A_n \downarrow A = \phi$.

$\mu(A_n) = \infty$ $\forall n \geq 1$ $\Rightarrow$ $\mu(A_n) \not\to \mu(A)$

$\mu(\phi) = 0$

$\square$

C-12 Probability Theory Semester I 2022/23 Quiz 1
https://www.isibang.ac.in/~athreya/Teaching/c12

1. Suppose μ is a measure defined on $(\mathbb{Z}, \mathcal{P}(\mathbb{Z}))$. Suppose $B_n \in \mathcal{P}(\mathbb{Z}), B_1 \supseteq B_2 \supseteq \ldots$ and $B = \cap_{n=1}^{\infty} B_n$ then is it true that $\mu(B) = \lim_{n\to\infty} \mu(B_n)$?

Solution: Let $\mu : \mathcal{P}(\mathbb{Z}) \longrightarrow [0,\infty) \cup \{\infty\}$ be given by

$\mu(A) = |A|$ (i.e. Counting measure)

Ex: Verify μ is a measure

$A_n = \{k \in \mathbb{Z} \mid k \geq n\}$

$A_n \supseteq A_{n+1}$ $\forall n \geq 1$ and $A_n \downarrow A = \phi$.

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