Due Date: 5th October 2022, 10pm

1. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a Probability space.Let Y be an integrable random variable on $(\Omega, \mathcal{F}, \mathbb{P})$. Using (post verifying all the hypotheses) Fubini's theorem show that

$$E[\mid Y \mid] = \int_0^\infty P[\mid Y \mid \geq t] dt$$

2. Exercise 9.5.14 [Ros06]

3. Let $\Omega_1 = \Omega_2 = \mathbb{N}$. Let $\mathcal{B}_1 = \mathcal{B}_2 = \mathcal{P}(\mathbb{N})$. Let $\mu_i, i = 1, 2$ be two measures on $(\Omega_i, \mathcal{B}_i)$ respectively defined by $\mu_1(\{n\}) = \mu_2(\{n\}) = \frac{1}{2^n}, \forall n \in \mathbb{N}$. Let $f : \Omega_1 \times \Omega_2 \to \mathbb{R}$ such that

$$f(m,n) = \begin{cases} n2^{2n} & \text{if } m = n,\, n \in \mathbb{N} \\ -n2^{2n-1} & \text{if } m = n-1,\, n \in \mathbb{N} \\ 0 & \text{otherwise} \end{cases}$$

   (a) Show that f is measurable with respect to the product σ-algebra $\mathcal{B}_1 \otimes \mathcal{B}_2$.

   (b) Discuss if Fubini's Theorem applies here.

References

[Ros06] Jeffrey S Rosenthal. *First Look At Rigorous Probability Theory, A.* World Scientific Publishing Company, 2006.