Due Date: 14th September 2022, 10pm *Problems Due: 2,5*

1. (*Book-Keeping*) Let $\mathbb{T}_2 = \{0\} \cup_{n=1}^{\infty} \{0,1\}^n$ be the binary tree. Show that for any $x, y \in \mathbb{T}_2$ there exists a unique point $c \equiv c(x, y, 0)$ such that $[x, 0] \cap [x, y] = \{0\}$ where $[x, 0]$ and $[x, y]$ denote the vertices on the unique path between $x, 0$ and x, y respectively.

2. At the Indian Statistical Institute to control the pace of walking among students several walk-traffic signals are placed Suppose the number of red signal Suppandi encounters while coming to C-12 on an average is 6 (according to MMath student class entry time database). Suppandi will be late if they encounter 8 or more red signals. Let X be the number of red signals on a given day.
   (a) Give a bound for $P(X \geq 8)$ using Markov inequality.
   (b) Suppose $\text{Var}[X] = 3$, give a bound for $P(X \geq 8)$ using TChebyschev's inequality.
   (c) Suppose $X \sim \text{Binomial}(12, 0.5)$, give a bound for $P(X \geq 8)$ using Chernoff bound and also provide the exact answer.
   (d) Compare and see which one gave the tightest bound.

3. Let $X \sim$ Binomial (n, p). Using the Chernoff bounds show that for $\delta > 0$
   (a) $P(X \geq (1+\delta)np) \leq \exp(-\frac{\delta^2(np)}{2+\delta})$
   (b) $P(X \leq (1-\delta)np) \leq \exp(-\frac{\delta^2(np)}{2})$

4. Let $X \sim$ Normal (μ, σ^2), show that for $c > 0$ $P(X - \mu \geq c) \leq \exp(-\frac{c^2}{2\sigma^2})$.

5. Let $\{A_n\}_{n\geq 1}$ be a sequence of pairwise independent events. For $m \geq 1$, let $X_m = \sum_{i=1}^{m} 1_{A_i}$.
   (a) Show that $P(X_m \geq 1) \geq \frac{1}{1+(\sum_{k=1}^{m} P(A_k))^{-1}}$
   (b) Using (a) show that if $\sum_{k=1}^{\infty} P(A_k) = \infty$ then $P(A_n$ occur i.o.$) = 1$