C-12 Probability Theory | Semester I 2022/23 | Homework 2

https://www.isibang.ac.in/~athreya/Teaching/c12

Due Date: 17th August 2022, 10pm *Problems Due: 2,4,5*

1. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a Probability space.and $\{Y_n\}$ be a sequence of random variables on $(\Omega, \mathcal{F}, \mathbb{P})$.
   (a) Show that $E = \{\omega \in \Omega : \exists Y(\omega) s.t. Y_n(\omega) \to Y(\omega)$ as $n \to \infty\}$ is a measurable set.
   (b) Show $\bar{Y} := \limsup_{n\to\infty} Y_n$ is a random variable.
   (c) Show $\underline{Y} := \liminf_{n\to\infty} Y_n$ is a random variable.
2. Exercise 2.7.14 in [Ros06]
3. Exercise 3.6.5 in [Ros06]
4. Exercise 3.6.7 in [Ros06]
5. Exercise 3.6.12 in [Ros06]

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1. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a Probability space.Suppose X, Y are independent random variables on $(\Omega, \mathcal{F}, \mathbb{P})$and $f : \mathbb{R} \to \mathbb{R}$ be a Borel-measurable function. Show $f(X)$ and $f(Y)$ are also independent.
2. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a Probability space.Suppose $A_n \in \mathcal{F}$ for all $n \geq 1$. Then show that

$$\mathbb{P}(\liminf_{n\to\infty} A_n) \leq \liminf_{n\to\infty} \mathbb{P}(A_n).$$

3. Exercise 1.2.10 in [AS08]
4. Exercise 1.2.11 in [AS08]
5. Exercise 1.3.6 in [AS08]
6. Exercise 1.4.9 in [AS08]

References

[AS08] Siva Athreya and Viakalathur Shankar Sunder. *Measure & probability*. Universities Press, 2008.

[Ros06] Jeffrey S Rosenthal. *First Look At Rigorous Probability Theory, A*. World Scientific Publishing Company, 2006.