Due: Tuesday December 8th 2020
Problem to be turned in None

1. Let $Y_1,\ Y_2,\ \cdots,\ Y_n$ be independent random variables, each uniformly distributed over the interval $(0, \theta)$.
   (a) Show that the mean $\bar{Y}$ converges in probability[1] towards a constant as $n \to \infty$ and find the constant.
   (b) Show that $\max\{Y_1, \cdots, Y_n\}$ converges in probability toward θ as $n \to \infty$.

2. The length of time (in appropriate units) that a certain type of component functions before failing is a random variable with probability density function
$$f(x) = \begin{cases} 2x & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$
   Once the component fails it is immediately replaced with another one of the same type.
   (a) If we let X_i denote the lifetime of the i^{th} component to be put in use, then $S_n = \sum_{i=1}^{n} X_i$ represents the time of the n^{th} failure. The long–term rate at which failures occur is
$$r = \lim_{n\to\infty} \frac{n}{S_n}$$
   Determine r, assuming that the random variables X_i are independent.
   (b) How many components would one need to have on hand to be approximately 90% certain that the stock would last at least 35 units of time?

3. Suppose S_n is binomially distributed with parameters $n = 200$ and $p = 0.3$ Use the central limit theorem to find an approximation for $P(99 \le S_n \le 101)$.

4. How often should you toss a coin:
   (a) to be at least 90 % sure that your estimate of the P(head) is within 0.1 of its true value ?
   (b) to be at least 90 % sure that your estimate of the P(head) is within 0.01 of its true value ?

5. A medical study is conducted to estimate the proportion of people suffering from April allergies in Bangalore. How many people should be surveyed to be at least 99% sure that the estimate is within 0.02 of the true value ?

[1] X_n converges in probability to X if for each $\epsilon > 0$, $\lim_{n\to\infty} P(\mid X_n - X \mid > \epsilon) = 0$.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2882	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3079	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3290	0.3315	0.3340	0.3365	0.3389
1.0	0.3414	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3622
1.1	0.3643	0.3665	0.3687	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4083	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4193	0.4207	0.4222	0.4237	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4358	0.4370	0.4382	0.4394	0.4406	0.4418	0.4430	0.4441
1.6	0.4452	0.4463	0.4474	0.4485	0.4495	0.4505	0.4516	0.4526	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4600	0.4608	0.4617	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4679	0.4686	0.4693	0.4700	0.4706
1.9	0.4713	0.4720	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4762	0.4767
2.0	0.4773	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4813	0.4817
2.1	0.4822	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4858
2.2	0.4861	0.4865	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4914	0.4916
2.4	0.4918	0.4920	0.4923	0.4925	0.4927	0.4929	0.4931	0.4933	0.4934	0.4936
2.5	0.4938	0.4940	0.4942	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4954	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4966	0.4967	0.4968	0.4969	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4975	0.4975	0.4976	0.4977	0.4978	0.4978	0.4979	0.4980	0.4980	0.4981
2.9	0.4982	0.4982	0.4983	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4988	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4991	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4993	0.4993	0.4993
3.2	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995	0.4995
3.3	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998	0.4998	0.4998

Table 1: Normal tables evaluating : $\frac{1}{\sqrt{2\pi}}\int_0^z e^{-\frac{x^2}{2}}\,dx$