Due: Tuesday Novembe 24th 2020

Problem to be turned in: 1, 2(e),f

1. Let X be a random variable with density $f(x) = 3x^2$ for $0 < x < 1$ (and $f(x) = 0$ otherwise). Calculate the distribution function of X.

2. Let $X \sim \text{Uniform}(0, 1)$.
   (a) Let $Y = \sqrt{X}$. Determine the density of Y.
   (b) Let $Z = \frac{1}{X}$. Determine the density of Z.
   (c) Let $r > 0$ and define $Y = rX$. Show that Y is uniformly distributed on $(0, r)$.
   (d) Let $Y = 1 - X$. Show that $Y \sim \text{Uniform}(0, 1)$ as well.
   (e) Let a and b be real numbers with $a < b$ and let $Y = (b - a)X + a$. Show that $Y \sim \text{Uniform}(a, b)$.
   (f) Find a function $g(x)$ (which is strictly increasing) such that the random variable $Y = g(X)$ has density $f_Y(y) = 3y^2$ for $0 < y < 1$ (and $f_Y(y) = 0$ otherwise).

3. Let $X \sim \text{Uniform}(\{1, 2, 3, 4, 5, 6\})$. Find the distribution function $F_X(x)$.

4. If $X \sim$ Normal $(0, 1)$. Let $Y = X^2$. Find the density function of Y