Due: Tuesday Novembe 17th 2020

Problem to be turned in: 4,6,9.

1. Let X and Y be discrete random variables with Range (X) $=\{0,1,2\}$ and Range $(Y) = \{1,2\}$ with joint distribution given by the chart below.

	$X=0$	$X=1$	$X=2$
$Y=1$	0.1	0.2	0.1
$Y=2$	0.3	0.2	0.1

   (a) Find $E[XY]$.

   (b) Compute $\text{Cov}(X,Y) := E[XY] - E[X]E[Y]$

2. Let X, Y be discrete random variables. Suppose $X \leq Y$ then show that $E[X] \leq E[Y]$.

3. Let $X \sim \text{Geometric}(p)$ and let A be event $(X \leq 3)$. Calculate $E[X|A]$ and $Var[X|A]$.

4. Let X and Y be described by the joint distribution

	$X=-1$	$X=0$	$X=1$
$Y=-1$	1/15	2/15	2/15
$Y=0$	2/15	1/15	2/15
$Y=1$	2/15	2/15	1/15

   and answer the following questions.

   (a) Calculate $E[X|Y=-1]$.

   (b) Calculate $Var[X|Y=-1]$.

   (c) Describe the distribution of $E[X|Y]$.

   (d) Describe the distribution of $Var[X|Y]$.

5. Let $f(x)$ be defined by

$$f(x) = \begin{cases} 2x & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Show that $f(x)$ is a probability density function.

   (b) Let P be the probability whose density is f. Calculate $P((0, \frac{1}{2}))$.

6. Let $f(x)$ be defined by

$$f(x) = \begin{cases} C \cdot sin(x) & \text{if } 0 < x < \pi \\ 0 & \text{otherwise} \end{cases}$$

   where C is a yet-to-be-determined constant.

(a) Determine the value of C that makes $f(x)$ a probability density function.

(b) Let P be the probability whose density if f. Calculate $P((0, \frac{1}{2}))$ and $P((\frac{1}{2}, 1))$.

(c) Which will be larger, $P((0, \frac{1}{4}))$ or $P((\frac{1}{4}, \frac{1}{2}))$? Explain how you can answer this question without actually calculating either probability.

(d) A game is played in the following way. A random variable X is selected with a density described by $f(x)$ above. You must select a number r and you win the game if the random variable results in an outcome in the interval $(r - 0.01, r + 0.01)$. Explain how you should choose r to maximize your chance of winning the game. (Very little computation should be required to answer this).

7. Let $X \sim \text{Exp}(\lambda)$. The "90th percentile" is a value a such that X is larger than a 90% of the time. Find the 90th percentile of X by determining the value of a for which $P(X < a) = 0.9$.

8. The "median" of a continuous random variable X is the value of x for which $P(X > x) = P(X < x) = \frac{1}{2}$.

   (a) If $X \sim \text{Uniform}(a, b)$ calculate the median of X.

   (b) If $Y \sim \text{Exp}(\lambda)$ calcluate the median of Y.

   (c) Let $Z \sim \text{Normal}(\mu, \sigma^2)$. Show that the median of Z is μ.

9. Let $X \sim \text{Normal}(\mu, \sigma^2)$. Show that $P(|X - \mu| < k\sigma)$ does not depend on the values of μ or σ.