Due: Thursday, Oct 27th 2020
Problem to be turned in: 4,5,8.

1. Let $i, j \in \mathbb{N}$. Show that the function $p(i,j) = \frac{1}{2^{i+j}}$ is a probability mass function. Now, consider X, Y be two random variables on a probabilty space $(\Omega, \mathcal{F}, P)$ with joint probability mass function given by $p(\cdot,\cdot)$. Compute $P(X+Y=4)$ and $P(X-Y=2)$.

2. Let X be the minimum and Y be the maximum of three digits picked at random wthout replacement from $\{0, 1, \ldots, 9\}$. Find the joint distribution of X and Y.

3. Of the people who enter a blood bank to donate blood, 1 in 3 have type O^+ blood and 1 in 15 have type O^- blood. For the next three people entering the blood bank, let X denote the number with type O^+ blood and Y the number with type O^- blood. Find the probability distributions for X, Y and $X+Y$.

4. Suppose that the number of earthquakes that occur in a year, anywhere in the world, is a Poisson random variable with mean λ. Suppose that the probability that any given earthquake has magnitude at least 5 on the Richter scale is p. Find the probability that there are n earthquakes with magnitude at least 5 in a year.

5. Suppose Ω is a sample space consisting of sequences of two coin flips. Let X be a r.v. that is 1 if the first coin is heads, and 0 otherwise, Y be a r.v. that is 1 if the first coin is tails, and 0 otherwise, and Z be a r.v. that is 1 if the second coin is tails, and 0 otherwise. (a) Show that X, Y, and Z all have the same p.m.f. (b) Show that the pairs (X,Y) and (X,Z) have different joint p.m.f.s. (c) Are X and Y independent? Why or why not? (d) Are X and Z independent? Why or why not?

6. Suppose tickets numbered $\{1, 2, \ldots, n\}$ are placed in a box and drawn one by one at random without replacement. Let X_i be the number of the ith ticket drawn, $1 \le i \le n$. (a) Find the joint distribution of $(X_1, X_2, \ldots, X_n)$. (b) Find the distribution X_j for $1 \le j \le n$.

7. Let X and Y be independent random variables each geometrically distributed with parameter p. (a) Find the distribution of $\min(X,Y)$. (b) Find $P(\min(X,Y) = X)$. (b) Find the distribution of $X+Y$. (c) Find the $P(X > m+n | X > n)$. (d) Find $P(Y = y | X+Y = z)$. r

8. Suppose X is a discrete random variable

   (a) If $E[X^2] < \infty$, then show that for any $b \in \mathbb{R}$, that $E|X-b|^2 < \infty$ and proceed to find b_0 such that
   $$E|X-b_0|^2 = \min_{b \in \mathbb{R}} E|X-b|^2.$$

   (b) If $E[X] < \infty$, then show that for any $b \in \mathbb{R}$, that $E|X-b| < \infty$ and proceed to find b_0 such that
   $$E|X-b_0| = \min_{b \in \mathbb{R}} E|X-b|.$$

9. Let m and r be positive integers and let N be an integer for which $N > \max\{m, r\}$. Let X be a random variable with $X \sim \text{HyperGeo}(N, r, m)$. Find $E[X]$ and $E[X^2]$.