

Due: Thursday, Oct 17th 2020

Problem to be turned in: 4,5,7.

1. Consider the experiment of flipping a coin four times and recording the sequence of heads and tails. Let S be the sample space of all sixteen possible orderings of the results. Let X be the function on S describing the number of tails among the flips. Let Y be the function on S describing the first flip (if any) to come up tails.
 - (a) Create a table as in Example 3.2.8 discussed in class, describing functions X and Y .
 - (b) Use the table to calculate $P(X = 2)$.
 - (c) Use the table to calculate $P(Y = 3)$.
2. A pair of fair dice are thrown. Let X represent the larger of the two values on the dice and let Y represent the smaller of the two values.
 - (a) Describe S , the domain of functions X and Y . How many elements are in S ?
 - (b) What are the ranges of X and Y . Do X and Y have the same range? Why or why not?
 - (c) Describe the distribution of X and describe the distribution of Y by finding the probability mass function of each. Is it true that $X \sim Y$?
3. A pair of fair dice are thrown. Let X represent the number of the first die and let Y represent the number of the second die.
 - (a) Describe S , the domain of functions X and Y . How many elements are in S ?
 - (b) Describe T , the range of functions X and Y . How many elements are in T ?
 - (c) Describe the distribution of X and describe the distribution of Y by finding the probability mass function of each. Is it true that $X \sim Y$?
 - (d) Are X and Y the same function? Why or why not?
4. Use the \sim notation to classify the distributions of the random variables described by the scenarios below. For instance, if a scenario said, “let X be the number of heads in three flips of a coin” the appropriate answer would be $X \sim \text{Binomial}(3, \frac{1}{2})$ since that describes the number of successes in three Bernoulli trials.
 - (a) Let X be the number of 5’s seen in four die rolls. What is the distribution of X ?
 - (b) Each ticket in a certain lottery has a 20% chance to be a prize-winning ticket. Let Y be the number of tickets that need to be purchased before seeing the first prize-winner. What is the distribution of Y ?
 - (c) A class of ten students is comprised of seven women and three men. Four students are randomly selected from the class. Let Z denote the number of men among the four randomly selected students. What is the distribution of Z ?
5. An urn has four balls labeled 1, 2, 3, and 4. A first ball is drawn and its number is denoted by X . A second ball is then drawn from the three remaining balls in the urn and its number is denoted by Y .
 - (a) Calculate $P(X = 1)$.
 - (b) Calculate $P(Y = 2|X = 1)$.
 - (c) Calculate $P(Y = 2)$.

- (d) Calculate $P(X = 1, Y = 2)$.
- (e) Are X and Y independent? Why or why not?
6. Two dice are rolled. Let X denote the sum of the dice and let Y denote the value of the first die.
- (a) Calculate $P(X = 7)$ and $P(Y = 4)$.
- (b) Calculate $P(X = 7, Y = 4)$.
- (c) Calculate $P(X = 5)$ and $P(Y = 4)$.
- (d) Calculate $P(X = 5, Y = 4)$.
- (e) Are X and Y independent? Why or why not?
7. Let X and Y be random variables with joint distribution given by the chart below.

	$X = 0$	$X = 1$	$X = 2$
$Y = 0$	$1/12$	0	$3/12$
$Y = 1$	$2/12$	$1/12$	0
$Y = 2$	$3/12$	$1/12$	$1/12$

- (a) Compute the marginal distributions of X and Y .
- (b) Compute the conditional distribution of X given that $Y = 2$.
- (c) Compute the conditional distribution of Y given that $X = 2$.
- (d) Carry out a computation to show that X and Y are not independent.