

**Due: Thursday, October 8th 2020**

*Problem to be turned in: 1,4,5.*

1. Two bookies, Gupta and Netika, bet Rupees 100 each on successive tosses of a coin. Each has Rupees 600 in hand.
  - (a) What is the probability that they break even after six tosses of the coin?
  - (b) What is the probability that Netika wins all of the money on the tenth toss of the coin?
2. Suppose that airplane engines operate independently in flight and fail with probability  $p$  ( $0 \leq p \leq 1$ ). A plane makes a safe flight if at least half of its engines are running. Kingfisher Air lines has a four-engine plane and Paramount Airlines has a two-engine plane for a flight from Bangalore to Delhi. Which airline has the higher probability for a successful flight ?
3. At Indian Statistical Institute, Bangalore there are 98 students. What is the probability that more than five students were born on Independence day ? Assume that birthrates are constant throughout the year and that each year has 365 days.
4. Suppose a fair coin is tossed 15 times. Find
  - (a)  $P(\{4 \text{ heads occur}\}|\{3 \text{ or } 4 \text{ heads occur}\})$ ,
  - (b)  $P(\{k-1 \text{ heads occur}\}|\{k-1 \text{ or } k \text{ heads occur}\})$ , and
  - (c)  $P(\{k \text{ heads occur}\}|\{k-1 \text{ or } k \text{ heads occur}\})$ .
5. Two intramural volleyball teams have eight players each. There is a 10% chance that any given player will not show up to a game, independently of any another. The game can be played if each team has at least six members show up. How likely is it the game can be played ?
6. Mark is a 70% free throw shooter. Assume each attempted free throw is independent of every other attempt. If he attempts ten free throws, answer the following questions.
  - (a) How likely is it that Mark will make exactly seven of ten attempted free throws?
  - (b) What is the most likely number of free throws Mark will make?
  - (c) How do your answers to (a) and (b) change if Mark only attempts 9 free throws instead of 10?
7. Continuing the previous exercise, Kalyani isn't as good a free throw shooter as Mark, but she can still make a shot 40% of the time. Mark and Kalyani play a game where the first one to sink a free throw is the winner. Since Kalyani isn't as skilled a player, she goes first to make it more fair.
  - (a) How likely is it that Kalyani will win the game on her first shot?
  - (b) How likely is it that Mark will win this game on his first shot? (Remember, for Mark even to get a chance to shoot, Kalyani must miss her first shot).
  - (c) How likely is it that Kalyani will win the game on her second shot?
  - (d) How likely is it that Kalyani will win the game?
8. A fair die is rolled repeatedly.
  - (a) What is the probability that the first 6 appears on the fifth roll?
  - (b) What is the probability that no 6's appear in the first four rolls?
  - (c) What is the probability that the second 6 appears on the fifth roll?