Due: Thursday, October 1st 2020
Problem to be turned in: 2,5,8.

1. Sheela has lost her key to her room. The security officer gives her 50 keys and tells her that one of them will open her room. She decides to try each key successively and notes down the number of the attempt at which the room opens. Describe the sample space for this experiment and also decide on the probability of each outcome.

2. (*Feller Volume 1 page 56, problem 20*) From a population of n elements a sample of size r is taken. Find the probability that none of the N prescribed elements will be included in the sample, assuming the sampling to be (a) without, (b) with replacement.

3. Suppose a population of size N contains G Green people and B Blue people, with $N = G + B$. A sample of size n, $1 \leq n \leq N$ is taken. Find the probability that g, $1 \leq g \leq \min(n, G)$, Green people are selected in the sample, assuming the sampling to be (a) without, (b) with replacement.

4. In a test called Narco-Analysis, a "truth" serum is given to a suspect. It is known that it is 90% reliable when the person is guilty and 99% reliable when the person is innocent. In other words 10% of the guilty are judged innocent by the serum and 1% of the innocent are judged guilty. If the suspect was selected from a group of suspects of which only 5% have ever committed a crime and the serum indicates that she is guilty, what is the probability that she is innocent ?

5. You first roll a fair die, then toss as many fair coins as the number that showed on the die. Given that 5 heads are obtained, what is the probability that the die showed 5 ?

6. *Polya Urn scheme*– An urn contains b black balls and r red balls. A ball is drawn at random. The ball is replaced into the urn along with c balls of its colour and d balls of the opposite colour. Then another random ball is drawn and the procedure is repeated.

   (a) What is the probability that the second ball drawn is a black ball ?
   (b) Assume $c = d$. What is the probability that the second ball drawn is a black ball ?
   (c) Assume $c = d$. What is the probability that the nth ball drawn is a black ball ?

7. A box contains M balls, of which W are white. A sample of n balls is drawn at random, **with** replacement. Let A_j, where $j = 1, 2, \cdots, n$, denote the event that the ball drawn on the jth draw is white. Let B_k denote the event that the sample of n balls contains exactly k white balls. Find $P(A_j|B_k)$.

8. It is estimated that 0.8% of a large shipment of eggs to a certain supermarket are cracked. The eggs are packaged in cartons, each with a dozen eggs, with the cracked eggs being randomly distributed. A restaurant owner buys 10 cartons from the supermarket.

   (a) If she notes the number of cartons containing cracked eggs, what are the possible outcomes for this experiment?
   (b) If she notes the total number of cracked eggs, what are the possible outcomes for this experiment?
   (c) How likely is it that she will find exactly one cracked egg among all of her cartons?
   (d) How likely is it that exactly one of her cartons will contain at least one cracked egg?
   (e) Explain why your answer to (d) is close to, but slightly larger than, than your answer to (c).
   (g) What is the most likely number of cracked eggs she will find among her cartons?
   (h) What is the most likely number of cartons that will contain at least one cracked egg?
   (i) How do you reconcile your answers to parts (g) and (h)?