1. Ismail and Dhamini are each dealt a 5 card hand from the same standard 52 card deck. What is the probability that they both hold "four of a kind" (Note: a player has four of a kind if he/she holds 4 two's, 4 three's, ..., 4 Kings or 4 Aces.)?

2. Suppose that n M.Math students attend a party and each student brings a gift. The gifts are placed in a box. The students stand in a line, and as they walk past the box they are handed a gift chosen at random from those remaining in the box. Let N be the number of students that receive the gift they brought with them.

   (a) Find $P\{N = n\}$.
   (b) Find $P\{N = n - 1\}$.
   (c) Find $E[N]$.
   (d) Find $\mathrm{Var}[N]$.

3. A box contains M balls, of which W are white. A sample of n balls, with $n \leq W$ and $n \leq M - W$, is drawn at random and **without** replacement. Let A_j, where $j = 1, 2, \cdots, n$, denote the event that the ball drawn on the j^{th} draw is white.

   (a) Find $P(A_1),\ P(A_2)$ and $P(A_3)$.
   (b) Find $P(A_j)$ for $1 \leq j \leq n$.

4. A box contains M balls, of which W are white. A sample of n balls is drawn at random, **with** replacement. Let A_j, where $j = 1, 2, \cdots, n$, denote the event that the ball drawn on the j^{th} draw is white. Let B_k denote the event that the sample of n balls contains exactly k white balls. Find $P(A_j|B_k)$.

5. A CRANE test given to a patient, for detecting CRANE-45 virus is known to be 90% reliable when the person has the disease and 99% reliable when the person does not have it. In other words, 10% of the patients who have the disease are judged to not have it and 1% of those who do not have it are judged to have the disease by test. If the patient was selected from a group of suspects of which only 5% have the disease and the test indicates that she has the disease, what is the probability that she does not ?

6. A random variable X has mean $\mu = 1$ and standard deviation $\sigma = 2$.

   (a) Find $E(2X + 1)$ and $V(2X + 1)$.
   (b) Estimate $P(-3 < X < 5)$ using Tchebysheff's theorem.

7. A book has 200 pages. The number of mistakes on each page is a Poisson random variable with mean 1, and is independent of the number of mistakes on all other pages.

   (a) What is the chance that there are at least 2 mistakes on the first page.
   (b) What is the chance that the first 5 pages has at least 2 mistakes in total.

8. At Blossom Babykutty Farms, a certain (random) number of eggs, N, is produced on a given day. Ms. Doddamagallu is considering two strategies for sending her eggs to market. Either she will put all her eggs in one basket (strategy 1), or she will put each egg in a separate basket (strategy 2). Any given basket has probability p of making it safely to market with all the eggs it contains intact; otherwise, all the eggs it contains are broken. Assume that the fates of the baskets sent off to market are independent of one another and independent of the total number of eggs, N. Let X (respectively, Y) be the total number of eggs that make it to market intact under strategy 1 (respectively, strategy 2).

(a) Show that $E[X] = E[Y] = pE[N]$.
(b) Show that $\text{Var}[X] = p\text{Var}[N] + p(1-p)(E[N])^2$.
(c) Show that $\text{Var}[Y] = E[N]p(1-p) + \text{Var}[N]p^2$.
(d) Conclude that $\text{Var}[X] \geq \text{Var}[Y]$, irrespective of the distribution of N.

9. Suppose two teams play a series of games, each producing a winner and a loser, until one team has won two more games than the other. Let G be the total number of games played. Assuming each team has chance 0.5 to win each game, independent of results of the previous games.

   (a) Find the probability function of G.
   (b) Find the expected value of G.

10. Let X be a normal random variable with mean -1 and standard deviation 2.

    (a) Find $P(X < -2)$.
    (b) Find the value of z_0 such that $P((X+1)^2 \leq z_0) = 0.3830$.

11. Continuous random variables X and Y have a joint density

$$f(x,y) = \begin{cases} k, & \text{for } 0 < x < 6, 0 < y < 4, \\ 0, & \text{elsewhere.} \end{cases}$$

    (a) Find k.
    (b) Find $P(2Y > X)$.
    (c) Find the marginal density of X.
    (d) Find the conditional density of Y given $X = 2$.
    (e) Are X and Y independent?
    (f) Find $E((X+Y)^2)$.

12. Let X and Y be two independent exponential random variables each with mean 1.

    (a) Find the density of $U_1 = X^{\frac{1}{2}}$.
    (b) Find the density of $U_2 = X + Y + 1$.
    (c) Find $P(\max\{X, Y\} > 1)$.

13. Suppose that U and V are independent random variables that both have the uniform distribution on $[0, 1]$.

    (a) Set $X = \max(U, V)$. What is the density of X?
    (b) Set $Y = V/U$. What is the density of Y?
    (c) What is the joint density of X and Y?
    (d) Are X and Y independent?

14. Suppose that X and Y are independent standard normal random variables. Set $V = \max(X, Y)$ and $W = \min(X, Y)$.

    (a) Find $\text{Var}[VW]$.
    (b) Express the probability $P\{W \geq V - 1\}$ in terms of Φ, the standard normal distribution function.

15. The length of time (in appropriate units) that a certain type of component functions before failing is a random variable with probability density function

$$f(x) = \begin{cases} 2x & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

    Once the component fails it is immediately replaced with another one of the same type. Assuming that each component is independent, how many components would one need to have on hand to be approximately 90% certain that the stock would last at least 35 units of time?