

- Concept of Independence · A, B - erents = independent = if occurrence of one erent does Not affect the probability of occurrence of the other · Generalize this to random variables

Conditional Probability

Consider the experiment of tossing a fair coin three times with sample space $S = \{hhh, hht, hth, htt, thh, tht, tth, ttt\}$. Let A be the event that there are two or more heads. As all outcomes are equally likely

$$P(A) = \frac{|\{hhh, hht, hht, hth, thh\}}{8}$$
$$= \cdot 4/8 = \sqrt{2}$$

Let B be the event that there is a head in the first toss. As above,

$$P(B) = \frac{|\{hhh, hht, hth, htt\}|}{8}$$
$$= \cdot 4/8 = \sqrt{2}$$

Now,

$$P(A|B) \leftarrow \frac{|A \cap B|}{|B|} = \frac{|A \cap B|}{|B|} = \frac{|A \cap B|}{|B|} = \frac{3}{4}$$

Let S be a sample space with probability P. Let A and B be two events with P(B) > 0. Then the conditional probability of A given B written as P(A|B) and is defined by

$$P(A|B) = rac{P(A \cap B)}{P(B)}.$$

- B has occurred - P(A(B) = Probability of A given that B hos occurred. Suppose we toss a coin three times. Then the sample space

$$S = \{hhh, hht, hth, htt, thh, tht, tth, ttt\}.$$
Let $A = \{first \text{ toss is head}\}$ and $B = \{scond \text{ toss is head}\}$

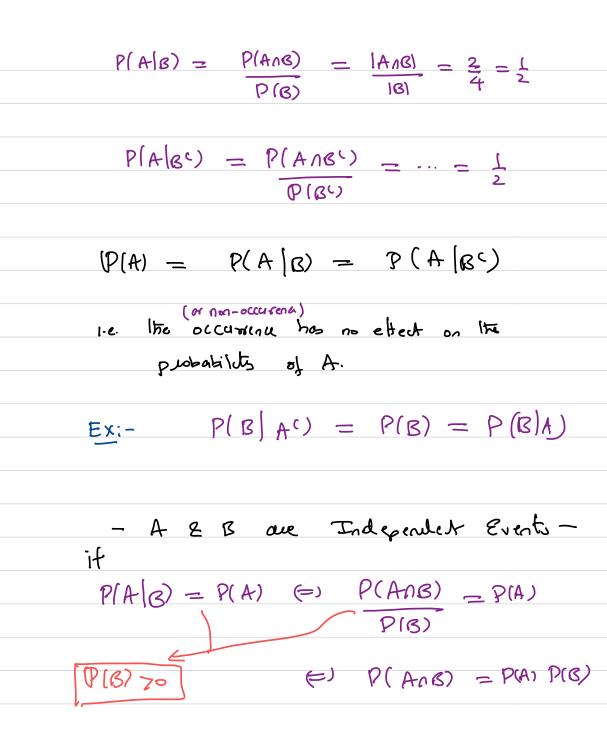
$$A = \{hhh, hht, hth, htt\} \text{ and } B = \{hhh, hht, thh, tht\}.$$

$$- \text{ Equally likely out core experiment}$$

$$P(A) = \frac{1}{2}$$

$$P(B) = \frac{1}{2}$$

$$P(A|B^{c}) = \frac{1}{2}$$
(See below)



Definition of Independence of Events :- :

• Two events A and B are independent if

$$P(A \cap B) = P(A)P(B).$$
 -) Equation

• A finite collection of events A_1, A_2, \dots, A_n is mutually independent if

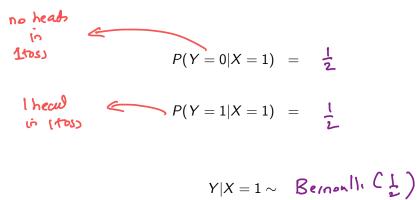
$$P(E_1 \cap E_2 \cap \cdots \cap E_n) = P(E_1)P(E_2) \dots P(E_n) - \int_{equation}^{2n} equation$$

whenever E_j is either A_j or A_j^c .

Definition of Independence for Random Variables:- :

- Two random variables X and Y are independent if (X ∈ A) and (Y ∈ B) are independent for every event A in the range of X and every event B in the range of Y
- A finite collection of random variables X₁, X₂,..., X_n is mutually independent if the sets (X_j ∈ A_j) are mutually independent for all events A_j in the ranges of the corresponding X_j.
- An arbitrary collection of random variables X_t where t ∈ I for some index set I is mutually independent if every finite sub-collection is mutually independent.

Let $X \sim \text{Uniform}(\{1,2\})$ and let Y be the number of heads in X tosses of a fair coin.



If X = 2 then Y is the number of heads in two flips of a fair coin then o head $P(Y=0|X=2) = V_{eq}$ 1 head $P(Y = 1 | X = 2) = V_2$ 2 heads P(Y=2|X=2) = 1/4

Therefore $Y|X = 2 \sim Bioonial (2, 5)$

 $\mathsf{Range}(X) = \{\iota, \mathbf{z}\} \text{ and } \mathsf{Range}(Y) = \{0, \iota, \mathbf{z}\}.$

To find the joint distribution of X and Y we must calculate the probabilities of each possibility. In this case the values may be obtained using the definition of conditional probability. For instance,

$$P(X = 1, Y = 0) = P(Y = 0 | X = 1) \cdot P(X = 1) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$$

and

$$P(X = 1, Y = 2) = P(Y = 2 | X = 1) \cdot P(X = 1) = 0 \cdot \frac{1}{2}$$

The entire joint distribution P(X = a, Y = b) is described by the following chart.

P(Y=x, Y=>)	X = 1	<i>X</i> = 2
<i>Y</i> = 0	1/4	1/8
Y = 1	1/4	1/4
Y = 2	0	1/8

Dependent Random Variables

There will be three different conditional distributions depending on
whether
$$Y = 0$$
, $Y = 1$, or $Y = 2$.
 $P(X = 1|Y = 0) = \frac{P(Y = 0|X = 1) \cdot P(X = 1)}{P(Y = 0)}$
(Ex to do
transle $P(Y = 0|X = 1) \cdot P(X = 1) = \frac{P(Y = 0|X = 1) \cdot P(X = 1)}{P(Y = 0)}$
 $= \frac{(Y_{\perp})(Y_{\perp})}{P(Y = 0|X = 1) \cdot P(X = 1)} \xrightarrow{P(X = L)} P(X = L)$
 $P(X = 2|Y = 0) = \dots$ Ex = $\dots = \frac{1}{3}$

Dependent Random Variables

	X = 1	X = 2	Sum	
Y = 0	1/4	1/8	3/8	-P(Y=3)
Y = 1	1/4	1/4	1/2	~P(Y=1)
Y = 2	0	1/8	V8	P(Y=L)
Sum	1/2	1/2		
	P(K=1)	(p (x=2)		

$$P(X = 1, Y = 0) = \bigvee_{4}$$
 while $P(X = 1) \cdot P(Y = 0) = \frac{1}{2} \cdot \frac{3}{8} = \frac{3}{12}$

3/16 = 1/4 =) × and Y are Not

$$\frac{(s^{1})^{s^{1}}}{(s^{1})^{s^{1}}} = \mathbb{P}(x=1)$$

$$\frac{1}{1} \mathcal{P}(x=1, Y=2)$$

$$= \mathbb{P}(x=1)$$

If X and Y are discrete random variables, the "joint distribution" of X and Y is the probability Q on pairs of values in the ranges of X and Y defined by

$$Q((a,b)) = P(X = a, Y = b).$$

The definition may be expanded to a finite collection of discrete random variables X_1, X_2, \ldots, X_n for which the joint distribution of all *n* variables is the probability defined by

$$Q((a_1, a_2, \ldots, a_n)) = P(X_1 = a_1, X_2 = a_2, \ldots, X_n = a_n).$$

Let X be a random variable on a sample space S and let $A \subset S$ be an event such that P(A) > 0. Then the probability Q described by

$$Q(B) = P(X \in B|A)$$

is called the "conditional distribution" of X given the event A.

Let $X : S \to T$ be a discrete random variable and let $A \subset S$ be an event for which P(A) > 0.

The "conditional expected value" of X given A is

$$E[X|A] = \sum_{t \in T} t \cdot P(X = t|A),$$

and the "conditional variance" of X given A is

$$Var[X|A] = E[(X - E[X|A])^2|A].$$

Recall X - Discrete 5.0Range (X) = T $E[X] = \sum_{t \in T} t P[X=6]$ $Var[X] = E[(X - E[X])^{2}]$

Example :-X~ Unifor (2), 23) Y~ Bioomial (X15) $E(Y | X=1) = \frac{1}{2}$ E(Y | X=z) = 1E[Y] = 1 P(Y=1) + 2 P(Y=2) $= 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{8}$ = ³/4