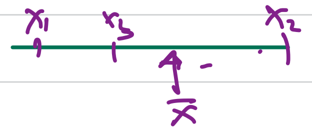
Recall :- $X : S \to T$ a discrete random variable

finite
infinite
may not exist $\leftarrow E[X] = \sum_{k \in T} k \, P(X=k)$

Table balances



weighted mean

"Spread of" R.V.

$$Var[X] = \sum_{k \in T} (k - E[X])^2 P(X=k)$$

"Range" of R.V

$$SD[X] = \sqrt{Var[X]}$$

$(-SD[X] + E[X], \; SD[X] + E[X])$

X ~ Binomial (n, p)

$E[X] = np$

Ex. $Var[X] = np(1-p)$

$SD[X] = \sqrt{np(1-p)}$

X ~ Geometric (p)

$E[X] = 1/p$

Ex: $Var[X] = ?$

Central limit Theorem :- [De-Moivre's]

- Very important result in Probability

- $X \sim$ Binomial (n, p), Range $(X) = \{0, .., n\}$

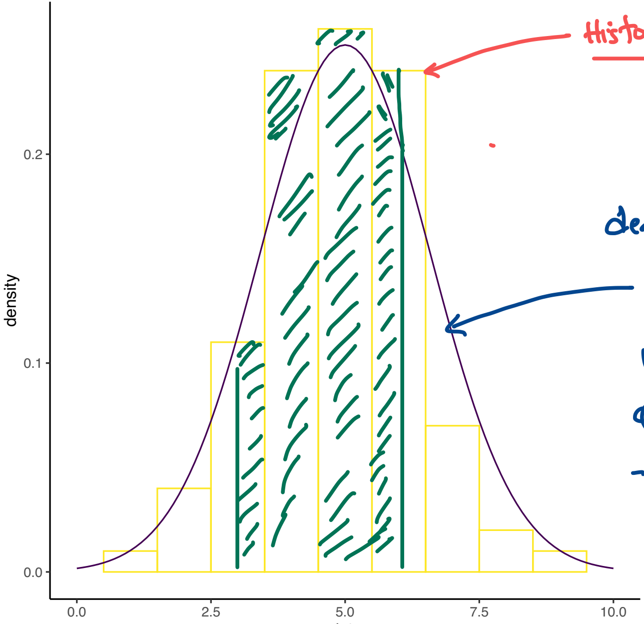
$a, b \in$ Range(X)

$$P(a < X \le b) = \sum_{k=a+1}^{b} P(X=k) = \sum_{k=a+1}^{b} \binom{n}{k} p^k (1-p)^{n-k}$$

Computational concerns $\equiv \dfrac{n(n-1)\cdots(n-k-1)}{k!}$ for large n and k

Binomial $(10, 0.5) \equiv b_1$

Revisit worksheet 3



Histogram of b1

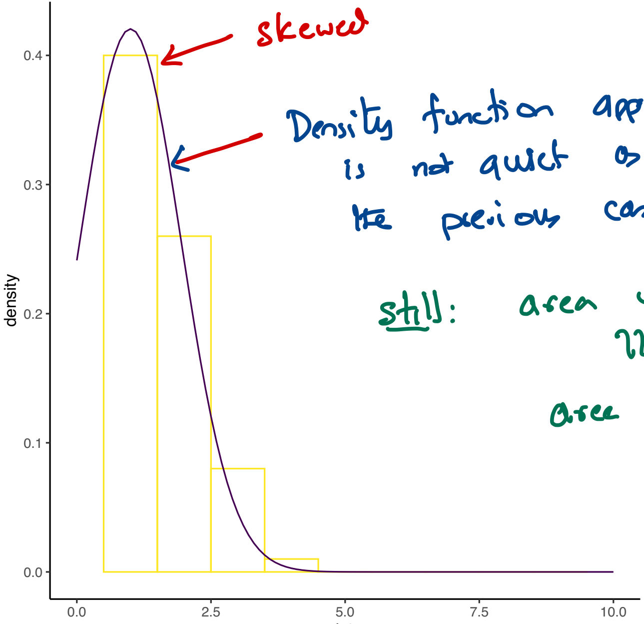
Q3 and Q2

$$\text{density}(x, 5, \sqrt{2.5}) = \frac{1}{\sqrt{2\pi\, 2.5}} e^{-\frac{(x-5)^2}{2(2.5)}}$$

Q 4

- Area under curve $\approx$ Area under Histogram

Binomial $(10, 0.1)$ $\approx$ b2



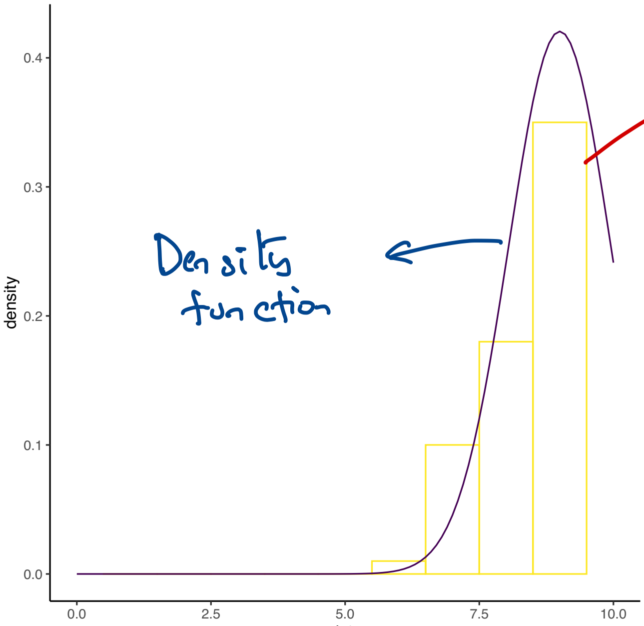
skewed

Density function approximation is not quite as good as the previous case

still: area under Histogram $\approx$ area under the curve.

Binomial $(10, 0.9)$ $\approx b_7$

skewed histogram



Density function

Central limit Theorem via Simulation

$X \sim$ Binomial (n, p) samples were generated in work sheet.

Q: $n \gg$ large $\quad \mathbb{P}(a < X \leq b) = ?$ ....

Is there a better way ?

Area under histogram between a and b

$\approx$

Area under the curve between a and b

$\Big\}$ work sheet 3 - $\Big\{$ Histogram, Curve

$$\mathbb{P}(a < X \leq b) \simeq \int_a^b \text{density}(x, \_, \_)\,dx$$

np

$\sqrt{np(1-p)}$

change of variable

$$\int_{*_2}^{*_1} \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}\,dx$$

Central limit Theorem :- let $0<p<1$ be fixed.

Let $X_n \sim \text{Binomial}(n, p) \quad \forall\, n \geq 1$

Let $\alpha, \beta \in \mathbb{R}$

$$\mathbb{P}\left(\alpha < \frac{X_n - np}{\sqrt{np(1-p)}} \leq \beta \right) \longrightarrow \int_{\alpha}^{\beta} \frac{e^{-\frac{y^2}{2}}}{\sqrt{2\pi}}\, dy$$

as $n \to \infty$.

Usage: $\mathbb{P}\left(\alpha\sqrt{np(1-p)} + np < X_n \leq \beta\sqrt{np(1-p)} + np \right)$

$$\approx \int_{\alpha}^{\beta} \frac{e^{-y^2/2}}{\sqrt{2\pi}}\, dy$$

Sums of Rolls