Please upload work sheet on moodle

Reference:- Chapter 1, 2, 3 in - Probability & statistics with examples using R.

Experiment 1 : Toss a coin 5 times

X = # of heads obtained.

Ω = set of all possible outcomes

$= \{ (H,H,H,H,H), (T,H,H,H,H), \ldots \ldots, \ldots (T,T,T,T,T) \}$

$\mathcal{F}$ = Events

$A = \{$we obtained 3 H in 5 Tosses$\}$

$\mathbb{P}$ - Probability of each event

$\mathbb{P}(A) = ?$

Mutually exclusive ways

$$\mathbb{P}(\underline{H}\ \underline{T}\ \underline{H}\ \underline{T}\ \underline{H}) = \left(\tfrac{1}{2}\right)^3 \left(\tfrac{1}{2}\right)^2 \qquad {}^5C_3$$

$$\mathbb{P}(A) = {}^5C_3 \left(\tfrac{1}{2}\right)^5$$

X - Random variable $X : \Omega \to \mathbb{R}$

$X(\{\omega_1, \omega_2, \ldots \omega_5\}) = \#\{i \mid \omega_i = H\}$

Rewrite: $A = \{X = 3\}$

(Sample Space)

Previous slide used Ω

- A sample space S is a set.
- The elements of the set S will be called "outcomes" and should be viewed as a listing of all possibilities that might occur.
- We will call the process of actually selecting one of these outcomes an "experiment".

$\mathcal{F} \equiv$ **(Temporary Definition of Event)** Given a sample space S, an "event" is any subset $E \subset S$. $\{E : E \subseteq S\} := \mathcal{F}$

- $A = \{(H, H, H, T, T), (H, T, H, T, H) \ldots (T, T, H, H, H)\}$

previous example.

(Probability Space Axioms)

set of all outcomes of Experiment

Recall :- Events

Let S be a sample space and let $\mathcal{F}$ be the collection of all events.
A "**probability**" is a function $P : \mathcal{F} \to [0, 1]$ such that

$P(S) = 1$; and

If $E_1, E_2, ...$ are a countable collection of disjoint events (that is, $E_i \cap E_j = \varnothing$ if $i \neq j$), then

$$P(\bigcup_{j=1}^{\infty} E_j) = \sum_{j=1}^{\infty} P(E_j). \tag{1}$$

$\Rightarrow$ A and B are disjoint event

$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$

Random Variable

Example :- $X : \Omega \to \mathbb{R}$ $\quad X = \#$ head in 5 tosses

- A "discrete random variable" is a function $X : S \to T$ where S is a sample space equipped with a probability P, and T is a countable (or finite) subset of the real numbers.
- P generates a probability on T and since it is a discrete space, the distribution may be determined by knowing the likelihood of each possible value of X.

$\boxed{\text{Range}(X) = T}$

- $f_X : T \to [0, 1]$ given by

$$f_X(t) = P(X = t)$$

referred to as a "probability mass function".

$T = \{0, 1, 2, 3, 4, 5\}$ $\quad \mathbb{P}(X = 3) = {}^5C_3 \left(\frac{1}{2}\right)^5$

Binomial Distribution

Experiment 1 : $X \sim$ Binomial $(5, \frac{1}{2})$

$X \sim$ **Binomial**(n, p): Let $0 \leq p \leq 1$ and let $n \geq 1$ be an integer. If X is a random variable taking values in $\{0, 1, \ldots, n\}$ having a probability mass function

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

for all $0 \leq k \leq n$, then X is a binomial random variable with parameters n and p. We have seen that such a quantity describes the number of successes in n Bernoulli trials.

Experiment 1 :

H = Success

T = failure