(Sample Space)

- A sample space S is a set.
- The elements of the set S will be called "outcomes" and should be viewed as a listing of all possibilities that might occur.
- We will call the process of actually selecting one of these outcomes an "experiment".

(Temporary Definition of Event) Given a sample space S, an "event" is any subset $E \subset S$.

(Probability Space Axioms)

Let S be a sample space and let $\mathcal{F}$ be the collection of all events.

A "**probability**" is a function $P : \mathcal{F} \to [0, 1]$ such that

$P(S) = 1$; and

If $E_1, E_2, ...$ are a countable collection of disjoint events (that is, $E_i \cap E_j = \varnothing$ if $i \neq j$), then

$$P(\bigcup_{j=1}^{\infty} E_j) = \sum_{j=1}^{\infty} P(E_j). \tag{1}$$

Random Variable

- A “discrete random variable” is a function $X : S \to T$ where S is a sample space equipped with a probability P, and T is a countable (or finite) subset of the real numbers.
- P generates a probability on T and since it is a discrete space, the distribution may be determined by knowing the likelihood of each possible value of X.
- $f_X : T \to [0, 1]$ given by

$$f_X(t) = P(X = t)$$

referred to as a “probability mass function”.

Binomial Distribution

$X \sim$ **Binomial**(n, p): Let $0 \leq p \leq 1$ and let $n \geq 1$ be an integer. If X is a random variable taking values in $\{0, 1, \ldots, n\}$ having a probability mass function

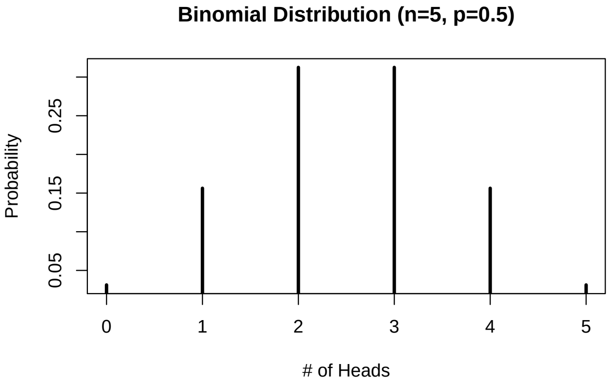
$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

for all $0 \leq k \leq n$, then X is a binomial random variable with parameters n and p. We have seen that such a quantity describes the number of successes in n Bernoulli trials.

Plotting Binomial Probabilities

```
> x <- 0:5
> plot(x,dbinom(x,size=5,prob=.5),
+      type='h',
+      main='Binomial Distribution (n=5, p=0.5)',
+      ylab='Probability',
+      xlab ='# of Heads',
+      lwd=3 )
```

Plotting Binomial Probabilities



Conditional Probability

Let S be a sample space with probability P. Let A and B be two events with $P(B) > 0$. Then the conditional probability of A given B written as $P(A|B)$ and is defined by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

X-outcome of fair die roll Y be the number of heads in X coin flips.

- What is the distribution of X?

X-outcome of fair die roll
Y be the number of heads in X coin flips.

- What is the distribution of $(Y|X = n)$ for $n = 1, \ldots, 6$?

X-outcome of fair die roll

Y be the number of heads in X coin flips.

- What is the conditional probability $P(X|Y=0)$?

Bayes Theorem

Theorem

Suppose A is an event, $\{B_i : 1 \leq i \leq n\}$ are a collection of disjoint events whose union contains all of A. Further assume that $P(A) > 0$ and $P(B_i) > 0$ for all $1 \leq i \leq n$. Then for any $1 \leq i \leq n$,

$$P(B_i|A) = \frac{P(A|B_i)P(B_i)}{\sum_{j=1}^{n} P(A|B_j)P(B_j)}.$$