Maximum likelihood Estimator :-

let $X_1, X_2, .. X_n$ be i.i.d $X <$ p.m.f. / p.d.f $f(x \mid p)$

$p \in P \subseteq \mathbb{R}^d$.

Likelihood function

$$L(p; X_1, .., X_n) = \prod_{i=1}^{n} f(X_i \mid p)$$

Suppose $\hat{p} \equiv \hat{p}(X_1, .., X_n)$ is a point at which $L(p; X_1, .., X_n)$ attains the maximum as a function of p

$\Rightarrow$ $\hat{p} \equiv$ maximum likelihood estimator of p (M.L.E.)

Applications to Edge Probability of Erdös-Renyi

Recall (Friday) Worksheet

(i) choose $x \in \{1, 2, 3, 4, 5\}$. $\equiv$ uniformly.

(ii) $A \equiv$ designate an event in the

Experiment of rolling a die, whose probability $p \equiv \frac{x}{6}$

[$X \equiv$ outcome of roll of a die

$\mathbb{P}(X=1) = 1/6$

$\mathbb{P}(X \leq 2) = 2/6 \quad \equiv x$

$\mathbb{P}(X \leq 3) = 3/6$

$\mathbb{P}(X \leq 4) = 4/6$

$\mathbb{P}(X \leq 5) = 5/6$ (E.g.)]

Review - Erdös - Renyi Graph $G(n, p)$

- late 40's - applications in networks
- well studied - simple to understand.

- $V = \{1, \ldots, n\}$ $\quad E \subseteq V \times V = \{\{i,j\} \mid i, j \in V\}$

- $i \sim j$ w.p. p

(place an edge with probability p)

- each vertex $i \in V \equiv$ can be connected to $n-1$ vertices

$$\boxed{\text{So there are } \frac{n(n-1)}{2} \text{ possible edges}}$$

Expected number of edges in Erdős Renyi :- $\frac{n(n-1)}{2} p$

(iii) Went to each edge $i \sim j$

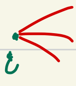
- Rolled the die
  - if event A occurred then placed an edge; otherwise not.

[Construction]

↳ graph and Adjacency matrix

Question:- From the graph $G(n, \frac{x}{6})$ can we estimate x? [$n=10$]

Answer:- let E be number of edges in $G(n, \frac{x}{6})$



i — $n-1$ edges w.p. p

- performing $\frac{n(n-1)}{2}$ trials with success probability $p = \frac{x}{6}$

- E = # of successes in $\frac{n(n-1)}{2}$ trials

$$E \sim \text{Binomial}\left(\frac{n(n-1)}{2}, \frac{x}{6}\right)$$

Likelihood for E

$$L(x;E) = \binom{\frac{n(n-1)}{2}}{E}\left(\frac{x}{6}\right)^{E}\left(1-\frac{x}{6}\right)^{\frac{n(n-1)}{2}-E}$$

$\therefore$ Given E :- Find x^* such that

$$L(x^*, E) = \max_{0 \le x \le 6} L(x;E)$$

Take logarithm

$$T(x) = \log(L(x;E)) = \log\binom{\frac{n(n-1)}{2}}{E} + E\log\left(\frac{x}{6}\right)$$

$$+ \left(\frac{n(n-1)}{2} - E\right)\log\left(1-\frac{x}{6}\right)$$

$$T'(x) = E \cdot \frac{6}{x} \cdot \frac{1}{6} + \left(\frac{n(n-1)}{2} - E\right)\frac{1}{1-\frac{x}{6}}\left(-\frac{1}{6}\right)$$

Set $T'(x) = 0 \Rightarrow$

$$\frac{E}{x} = \left(\frac{n(n-1)}{2} - E\right)\frac{1}{6}\cdot\frac{1}{1-\frac{x}{6}}$$

$$\Rightarrow \frac{1-\frac{x}{6}}{\frac{x}{6}} = \frac{n(n-1)}{2E} - 1$$

$$\Rightarrow \frac{1}{x/6} = \frac{n(n-1)}{2E}$$

$$\Rightarrow \boxed{x = \frac{6\times 2\times E}{n(n-1)}}$$

$\therefore$ $x^* = \dfrac{12E}{n(n-1)}$ is MLE for x in $G(n, \frac{x}{6})$

if there are E - edges.

E.g:- Suppose $E = 7$, $n = 10$ $\Rightarrow x^* = \dfrac{84}{90} = \dfrac{14}{15} \approx 1$

Estimate for $x = 1$.

Question 2 :- Consider $G(n, \frac{\lambda}{n})$ for $\lambda > 0$.

and n - large.

- let $d_n \equiv$ degree of a uniformly chosen vertex.

- What is $\lim_{n \to \infty} \mathbb{P}(d_n = k) \equiv$?

Veritasium

- σ - known $\quad \bar{X} - \bar{Y}$ - understood
- σ - unknown $\quad \dfrac{\bar{X} - \mu}{S}$ ✓ t $\quad \Big| \quad \dfrac{\bar{X} - \bar{Y}}{S}$ ..

$S \leftarrow$ unknown by estimate

II

statistic - Distribution

Data normal

Derived

distribution under null hypothesis

$\bar{X}$ - Derived

X ~ Normal

$\dfrac{\bar{X} - \bar{Y}}{S} \stackrel{d}{=}$ does not depend unknown σ

...

μ

z_i .. z_n .. $\dfrac{\bar{Z} - \bar{W}}{S} \leftarrow$ d.s

$\sigma = 1$, Fix any Simulate

E.g $\dfrac{U(-1,1) * S}{U(-1,1) \; \tilde{S}} \leftarrow \mu_1 \neq \mu_2$

E.C.DF (..) .. CDF.