Recall :-

Simulate law of large numbers

$X_1, \ldots, X_n$ i.i.d X $\quad E[X] = \mu \quad Var[X] = \sigma^2$

$$\bar{X}_n = \frac{\sum_{i=1}^{n} X_i}{n}$$

[WLLN]

$$\mathbb{P}(|\bar{X}_n - \mu| > \varepsilon) \longrightarrow 0 \quad \text{as} \quad n \to \infty$$

[SLLN]

$$A = \{\lim_{n\to\infty} \bar{X}_n = \mu\}$$

$$\Rightarrow \quad \mathbb{P}(A) = 1.$$

- $E\bar{X}_n = \mu \qquad Var(\bar{X}_n) = \frac{\sigma^2}{n}$

[CLT]

$$\sqrt{n}\left(\frac{\bar{X}_n - \mu}{\sigma}\right) \sim N(0,1)$$

→ fluctuations around μ

[C.I]

$$\mathbb{P}\left(\mu \in \left(\bar{X} - 1.96\frac{\sigma}{\sqrt{n}},\ \bar{X} + 1.96\frac{\sigma}{\sqrt{n}}\right)\right) \approx 0.95$$

Law of Large Numbers

```
> runningmean = function (x,N){
+ y = sample(x,N, replace=TRUE)
+ c = cumsum(y)
+ n = 1:N
+ c/n
+ }
> u = runningmean(c(0,1), 1000)
```

Sampling N points from {0,1} uniformly

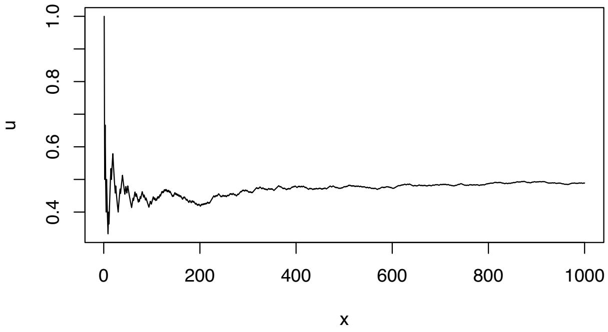
$c = (y_1, \dots, \sum_{i=1}^{k} y_i, \dots)$

$(1, 2, \dots k, \dots, N)$

$u = (\bar{X}_1, \bar{X}_2, \dots, \bar{X}_N)$

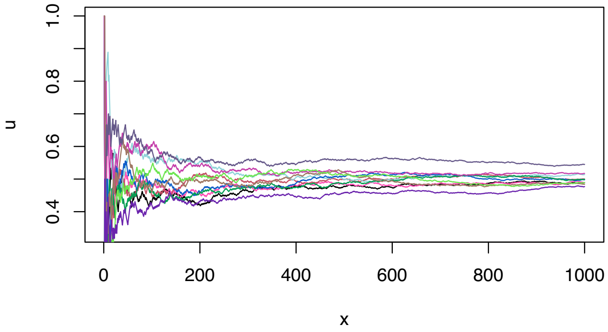
Law of Large Numbers

```
> x=1:1000; plot(u~x, type="l");
>
```



Law of Large Numbers

```
> x=1:1000; plot(u~x, type="l");
> replicate(10, lines(runningmean(c(0,1), 1000)~x, type="l",  col=rgb(runif(3),runif(3),runif(3))))
```



Law of Large Numbers