1. (Tschebychev Inequality)

   (a) Find a random variable X with Range$(X) = \{-1, 0, 1\}$ such that

   $$P(\mid X - \mu \mid \geq 2\sigma) = \frac{1}{4},$$

   with $\mu = E[X]$ and $\sigma^2 = \text{Var}[X]$.

   (b) Construct another random variable Y (different from X) with Range $(Y) = \{y_1, y_2, y_3\}$, mean μ and with

   $$P(\mid Y - \mu \mid > 2\sigma) > P(\mid X - \mu \mid > 2\sigma),$$

   so as to get

   $$P(\mid Y - \mu \mid > 2\sigma) > \frac{1}{4}$$

   Decide whether Tschebychev Inequality is violated ?

   (c) Write an R-code that takes an input k, and constructs a random variable X with Range$(X) = \{-1, 0, 1\}$ such that

   $$P(\mid X - \mu \mid \geq k\sigma) = \frac{1}{k^2},$$

   with $\mu = E[X]$ and $\sigma^2 = \text{Var}[X]$. Further the R-code should construct a random variable Y (different from X) with Range $(Y) = \{y_1, y_2, y_3\}$, mean μ so that

   $$P(\mid Y - \mu \mid > k\sigma) > \frac{1}{k^2}$$

   and (using replications) verifies your conclusion about Tschebychev's inequality in (b). It should save the entire output as a (suitably designed) csv file.

Due date: November 19th, 2021
Problems Due:1,3,5,7

From Probabiliy and Statistics with Examples using R.

1. Ex 3.2.4
2. Ex 3.2.5
3. Ex 3.2.9
4. Ex 3.3.7
5. Ex 3.3.11
6. Ex 3.3.15
7. Ex 4.4.3
8. Ex 4.4.4