Prashant Sharma

Grading:
20 marks- Complete submission of worksheet7
80 marks- Problem 1

Solution-1(a):
Given a random variable X with $Range(X)$=$\{-1,0,1\}$ such that

$$P(|X-\mu| \geq 2\sigma) = \frac{1}{4}....(1)$$

with $\mu = E[X]$ and $\sigma^2 = Var[X]$
Let $P(X=-1) = P(X=1) = \alpha$
Thus,

X	-1	0	1
$P(X=x)$	α	$1-2\alpha$	α

Now, $\mu = E[X] = (-1)\times\alpha + 0\times(1-2\alpha) + 1\times\alpha = 0$
$\sigma^2 = Var[X] = E[X^2] - (E[X])^2 = E[X^2] - 0 = E[X^2] = (-1)^2\times\alpha + 0^2\times(1-2\alpha) + 1^2\times\alpha = 2\alpha$
On putting the values of μ and σ in equation(1), we get

$$P(|X-0| \geq 2\sqrt{2\alpha}) = \frac{1}{4}$$
$$P(|X| \geq 2\sqrt{2\alpha}) = \frac{1}{4}$$

If, $2\sqrt{2\alpha} = 1$ i.e., $\alpha = \frac{1}{8}$, then condition satisfies.
Therefore,

X	-1	0	1
$P(X=x)$	$\frac{1}{8}$	$\frac{6}{8}$	$\frac{1}{8}$

with $\mu = E[X] = 0$ and $\sigma^2 = 2\alpha = 2\times\frac{1}{8} = \frac{1}{4}$
Hence, X is a random variable that satisfies the condition $P(|X-\mu| \geq 2\sigma) = \frac{1}{4}$

Solution-1(b):
Let us denote another random variable Y(different from X) with range(Y)= $\{y_1, y_2, y_3\}$ and with mean μ
Now,

$$P|Y-\mu| > k\sigma) > \frac{1}{k^2}...(2)$$

$E[Y] = \mu = E[X] = 0$ and $\sigma^2 = Var[X] = \frac{1}{k^2}$
On putting the value of μ and σ in equation(2),

$$P|Y| > 1) > \frac{1}{k^2}$$

To satisfy above condition we may consider the random variable Y such that the pmf of Y is:

X	-2	0	2
$P(Y = y)$	$\frac{1}{k^2}$	$1 - \frac{2}{k^2}$	$\frac{1}{k^2}$

provided $k^2 < 2$
Here it is given that $k = 2$, we get

X	-2	0	2
$P(Y = y)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

with $\mu = E[Y] = 0$ and $Var[Y] = 2$
Thus, Y is a random variable that satisfies the condition:

$$P|Y - \mu| > 2\sigma) > \frac{1}{4}$$

(By taking $\sigma = \frac{1}{2}$)

$$P(|Y - \mu| > 2\sigma) = P(|Y| > 1) = P(Y = -2) + P(Y = 2) = \frac{1}{2}$$

And,

$$P(|X - \mu| > 2\sigma) = P(|X| > 1) = 0$$

Hence,

$$P(|Y - \mu| > 2\sigma) > P(|X - \mu| > 2\sigma)$$

Tchebychev inequality is not violated since for random variable Y, we are using variance of random variable X not of Y.

Solution-1(c):
First, we perform the computation process used in part(a) to create the required probability distribution of x.

```
> fx=function(x,k)
+ {
+   if(x==0){
+     return(1-(1/(k^2)))
+   }
+   else if(abs(x)==1){
```

```
+     return(1/(2*(k^2)))
+   }
+   else
+     return(0)
+ }
> fx(-1,2)
```

```
[1] 0.125
```

```
> fx(0,2)
```

```
[1] 0.75
```

```
> fx(1,2)
```

```
[1] 0.125
```

Now, the probability distribution of Y is computed using the process according to part(b).

```
> fy=function(y,k){
+   if(y==0){
+     return(1-(2/(k^2)))
+   }
+   else if(abs(y)==2){
+     return(1/(k^2))
+   }
+   else
+     return(0)
+ }
> fy(-2,2)
```

```
[1] 0.25
```

```
> fy(0,2)
```

```
[1] 0.5
```

```
> fy(2,2)
```

```
[1] 0.25
```

```
> Tchebychev=function(k){
+   range_y=c(-2,0,2)
+   prob_y=c(0,0,0)
+   prob_y[1]=(1/(k^2))
+   prob_y[2]=(1-(2/(k^2)))
+   prob_y[1]=(1/(k^2))
+   prob_y
+   sample=sample(range_y,10000,replace=TRUE,prob=prob_y)
+   sample
+   prob=length(sample[abs(sample)>1])/10000
+   return(prob)
+ }
> Tchebychev(2)
```

```
[1] 0.3345
```

Now,

```
> M=matrix(0,ncol=10,nrow=99)
> df=data.frame(M)
> df[,1]=2:100
> for(i in 1:99){
+   df[i,2]=fx(-1,(i+1))
+ }
> for(i in 1:99)
+ {
+   df[i,3]=fx(0,(i+1))
+ }
> for(i in 1:99)
+ {
+   df[i,4]=fx(1,(i+1))
+ }
> for(i in 1:99)
+ {
+   df[i,5]=fy(-2,(i+1))
+ }
> for(i in 1:99)
+ {
+  df[i,6]=fy(0,(i+1))
+ }
> for(i in 1:99)
+ {
+  df[i,7]=fy(2,(i+1))
+ }
> for(i in 1:99)
+ {
+  df[i,8]=Tchebychev(i+1)
```

```
+ }
> for(i in 1:99){
+   df[i,9]=1/((i+1)^2)
+ }
> for(i in 1:99){
+   if(df[i,8]>df[i,9])
+     df[i,10]="Satisfied"
+   else
+     df[i,10]="Not Satisfied"
+ }
> colnames(df)=c("k","P(X=-1)","P(X=0)","P(X=1)","P(Y=-2)","P(Y=0)","P(Y=2)","P(Tchebychev Event)",
+                "Bound","Validation")
> ##Now, converting into csv format.
> write.csv(df,"C:\\Users\\shiva\\Desktop\\Bayesian Inference\\Worksheet7sol.csv")
```