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Problem 1:
Given, X be a Normal Random variable with mean $\mu = 3$ and standard deviation $\sigma = 1$.

1(a):

P(2.1 < X < 3.4)= P(X<3.4)-P(X<2.1)

$$= P(\frac{X-3}{1} < \frac{3.4-3}{1}) - P(\frac{X-3}{1} < \frac{2.1-3}{1})$$

As we know that $\frac{X-\mu}{\sigma} = Z$ which is a standard normal variable. Therefore,

$$= P(Z < 0.4) - P(Z < -0.9)$$

$$= P(Z < 0.4) - (1 - P(Z < 0.9)$$

$$= P(Z < 0.4) - 1 + P(Z < 0.9)$$

On putting values from standard normal table, we get

$$= 0.6554 - 1 + 0.8159$$

$$= 0.4713$$

Thus, $P(2.1 < X < 3.4) = 0.4713$

1(b):

```
> #Calculation of P(2.1 < X < 3.4) by using pnorm() function
> pnorm(3.4,3,1)-pnorm(2.1,3,1)
```

```
[1] 0.4713616
```

Problem 2:

Given, X be a Normal Random variable with mean $\mu = 3$ and standard deviation $\sigma = 4$.

2(a):

$P(|X-\mu| < \sigma) = P(-1 < \frac{X-\mu}{\sigma} < 1) = P(-1 < Z < 1) = P(Z < 1) - P(Z < -1)$
$= P(Z < 1) - 1 + P(Z < 1) = 2P(Z < 1) - 1$

```
> #Calculation by using pnorm() function
> 2*pnorm(1,0,1)-1
```

```
[1] 0.6826895
```

Thus, $P(|X-\mu| < \sigma) = 0.6826895$.

2(b):

$P(|X-\mu| < 2\sigma) = P(-2 < \frac{X-\mu}{\sigma} < 2) = P(-2 < Z < 2) = P(Z < 2) - P(Z < -2)$
$= P(Z < 2) - 1 + P(Z < 2) = 2P(Z < 2) - 1$

```
> 2*pnorm(2,0,1)-1
```

```
[1] 0.9544997
```

Thus, $P(|X-\mu| < 2\sigma) = 0.9544997$.

2(c):

$P(|X-\mu| < 3\sigma) = P(-3 < \frac{X-\mu}{\sigma} < 3) = P(-3 < Z < 3) = P(Z < 3) - P(Z < -3)$
$= P(Z < 3) - 1 + P(Z < 3) = 2P(Z < 3) - 1$

```
> 2*pnorm(3,0,1)-1
```

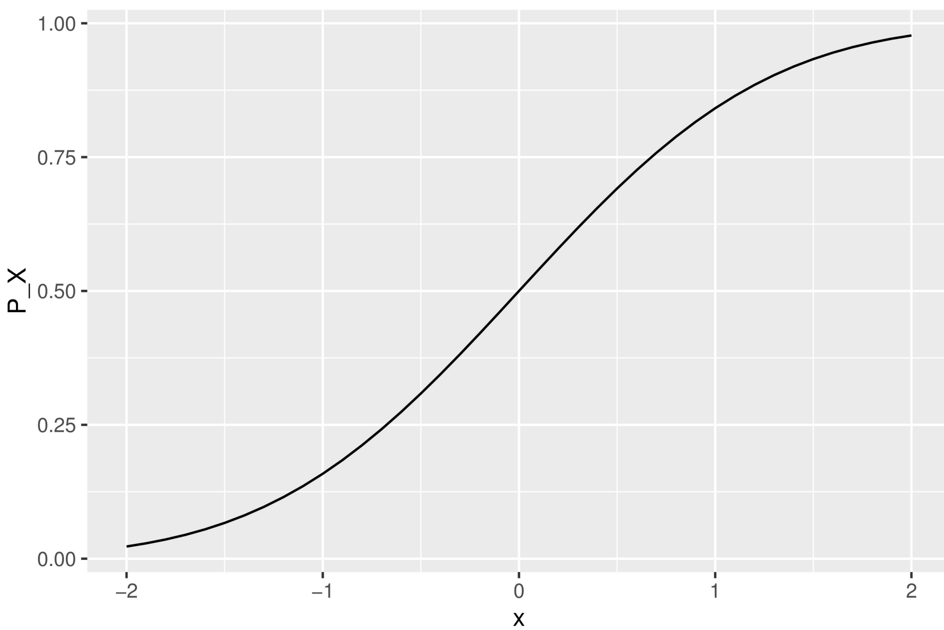
```
[1] 0.9973002
```

Thus, $P(|X-\mu| < 3\sigma) = 0.9973002$.

Problem-3:

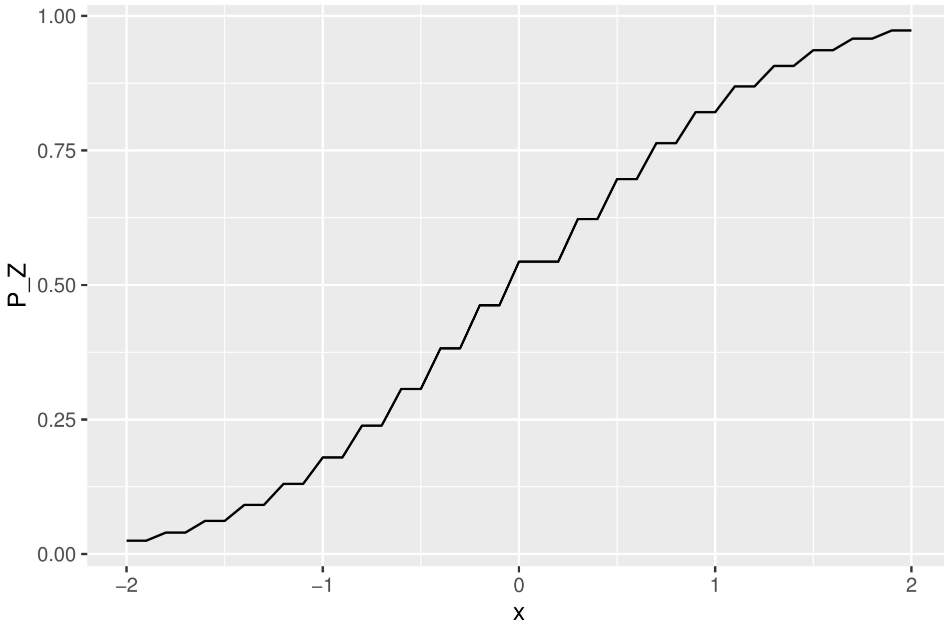
3(a):

```
> library(ggplot2)
> x = seq(-2,2,0.1)
> P_X = pnorm(x)
> df = data.frame(P_X,x)
> Dist_funcn_plot=ggplot(df)+geom_line(aes(x=x, y=P_X))
> Dist_funcn_plot
```




3(b):

```
> m = 100
> p = 0.4
> #z = (Y - m*p)/((m*p*(1-p))^(0.5))
> Y = x*(m*p*(1-p))^(0.5) + m*p
> P_Z = pbinom(Y,m,p)
> df1=data.frame(P_Z,Y)
> Dist_funcn_plot_2=ggplot(df1)+geom_line(aes(x=x, y=P_Z))
> Dist_funcn_plot_2
```




Problem(4):

```
> b = array()
> c=1
> for(i in 0:2){
+   c=c*(10**i)
+   for(j in 1:100){
+     a= rexp(5*c,1)
+     b[j]=mean(a)
+   }
+   print("sample mean for 100 trails of n=")
+   print(5*c)
+   print("are")
+   print(b)
+   print("average value and variance of the sample mean are:")
+   print(mean(b))
+   print(var(b))
+ }
```

```
[1] "sample mean for 100 trails of n="
[1] 5
[1] "are"
  [1] 0.9698709 0.6472605 1.0630121 0.8974616 1.3964661 0.8872318 0.7145940
  [8] 0.6305584 1.4237100 1.0785249 1.3186422 1.3991485 0.9408413 0.8339773
 [15] 2.6378716 0.8061330 1.1917127 1.0432840 1.2651410 1.7805236 1.0575389
 [22] 0.1657503 0.8196603 1.1216141 0.5479102 0.9589994 0.5258797 0.2660520
 [29] 0.7766919 0.4884298 3.3425458 1.5590116 1.5552526 1.5004313 0.2820485
 [36] 0.6668143 1.2329213 1.6727935 0.7267764 0.8709976 0.7208827 1.9067585
 [43] 2.1637267 0.8419789 0.8998943 1.1243344 0.5324144 0.9790428 0.7770938
 [50] 1.4564871 0.9940447 0.7271780 0.4433604 0.5916793 1.0426077 1.8087952
 [57] 0.7318909 0.8035596 1.2385078 0.8653515 0.9763260 2.2805106 1.8897214
```

```
 [64] 0.4078913 1.2637978 0.4065090 0.7726391 1.0303816 1.0523249 1.1509693
 [71] 1.0708185 1.2976348 1.2730873 0.9163698 0.7092458 0.9923909 0.4446327
 [78] 0.5114334 0.5435837 1.2706010 0.5137228 1.6521387 1.0076625 0.5634036
 [85] 0.3889796 0.8672523 0.6644745 0.3720281 0.9318883 0.9892016 1.3803596
 [92] 1.0062142 1.1460045 1.1363021 0.7247978 1.5990166 0.5469707 1.5554177
 [99] 0.9180076 0.7409269
[1] "average value and variance of the sample mean are:"
[1] 1.026793
[1] 0.2574757
[1] "sample mean for 100 trails of n="
[1] 50
[1] "are"
  [1] 0.9453345 1.0564534 0.9856723 1.3249806 1.0178280 0.9509542 0.7846652
  [8] 0.8753245 1.0112455 1.1103634 0.9651951 0.8170279 1.2361274 1.1718415
 [15] 1.1161733 1.0639459 0.8530072 1.1014534 1.0522937 0.9879602 0.9793563
 [22] 1.2083715 1.1798565 0.9239524 0.7279536 0.9519226 0.9403709 0.9153879
 [29] 1.0165118 1.1551659 0.8449005 0.7952029 0.9424086 0.9088906 0.7969058
 [36] 1.0002871 0.8849574 1.0656891 1.0272645 1.0341782 0.8645461 1.0824960
 [43] 0.9955068 1.0963104 0.9515671 0.9356857 0.9133295 0.8829163 0.7159052
 [50] 0.7634117 1.0136916 0.9877268 1.0690805 1.1059542 1.0587391 1.0830452
 [57] 0.8764405 0.8838499 0.8747503 1.1391573 1.0089654 0.9763808 0.9551747
 [64] 1.0858719 0.9207993 0.8453924 0.8141436 0.9761129 0.7658642 0.8232944
 [71] 0.7003730 0.9916108 1.2637825 1.3576692 1.1056220 1.1369547 1.2992870
 [78] 0.9371186 1.1147078 1.1814773 1.4209171 1.1262357 0.9919703 0.9582932
 [85] 1.0178309 1.1312528 1.0466851 0.8879783 0.7964076 0.9738853 1.0478720
 [92] 1.1792633 0.8778488 1.2071699 1.0418393 1.2971754 1.0291430 0.8173053
 [99] 0.8117997 0.8430070
[1] "average value and variance of the sample mean are:"
[1] 0.9978597
[1] 0.02167691
[1] "sample mean for 100 trails of n="
[1] 5000
[1] "are"
  [1] 0.9748518 0.9907299 0.9915381 1.0113368 0.9891010 0.9735558 0.9982271
  [8] 0.9973368 1.0117087 0.9909002 0.9890529 0.9910525 0.9877319 1.0088486
 [15] 0.9786788 1.0122446 1.0078030 1.0067893 0.9999715 1.0095926 0.9938490
 [22] 1.0060693 0.9869370 1.0008548 1.0040038 0.9958147 1.0077116 1.0026657
 [29] 1.0129130 0.9699116 0.9957110 1.0015161 1.0062088 1.0244098 1.0010454
 [36] 1.0034239 1.0150063 1.0256574 0.9916465 0.9974176 0.9940431 1.0083573
 [43] 1.0160884 1.0300073 0.9775273 0.9999760 0.9843413 0.9882788 0.9832050
 [50] 1.0000969 1.0054956 1.0189692 0.9750696 0.9876323 0.9953680 0.9884197
 [57] 0.9928072 1.0195243 1.0170032 0.9847195 0.9907714 0.9888195 1.0172239
 [64] 1.0259703 1.0091133 0.9998450 0.9855188 1.0089821 0.9922142 0.9812670
 [71] 1.0116055 0.9790125 0.9891118 0.9919238 1.0018140 0.9902957 0.9998716
 [78] 0.9913414 1.0048795 0.9928721 1.0145518 1.0010716 0.9984598 0.9824036
 [85] 0.9836517 0.9889477 0.9844094 1.0129819 0.9955384 0.9850551 0.9778102
 [92] 1.0042345 0.9952627 0.9846795 1.0118302 0.9804436 1.0167072 1.0159563
 [99] 1.0043489 1.0104314
[1] "average value and variance of the sample mean are:"
[1] 0.9983198
[1] 0.0001733881
```

On comparing the average value and variance for each case, we observe that average value tends to the mean of Exp(1) and variance goes to decrease as sample size increases.