

For numerical answers with decimal digits please read instructions.

1. Let X, Y be two discrete random variables taking values $\{-1, 1\}$. Suppose their joint distribution is given by the table

	$X=-1$	$X=1$
$Y=-1$	0.3	0.2
$Y=1$	0.3	0.2

- (a) $Cov(X, Y)$ is ____.
- (b) Are X and Y independent ?
2. Let X and Y be discrete random variables with $\text{Range}(X) = \{0, 1, 2\}$ and $\text{Range}(Y) = \{1, 2\}$ with joint distribution given by the chart below.

	$X = 0$	$X = 1$	$X = 2$
$Y = 1$	0.1	0.2	0.1
$Y = 2$	0.3	0.2	0.1

- (a) $\mathbb{E}[XY]$ is $\frac{(i)}{(ii)}$.
The above fraction should be in the simplest form, i.e. $\text{g.c.d} \{ (i), (ii) \} = 1$.
- (b) $\text{Cov}(X, Y) := \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$ is $\frac{(i)}{(ii)}$.
The above fraction should be in the simplest form, i.e. $\text{g.c.d} \{ (i), (ii) \} = 1$.
3. Let $X \sim \text{Geometric}(\frac{1}{2})$ and consider the event $A = \{X \leq 3\}$.
- (a) $\mathbb{E}[X|A]$ is $\frac{(i)}{(ii)}$.
The above fraction should be in the simplest form, i.e. $\text{g.c.d} \{ (i), (ii) \} = 1$.
- (b) $\text{Var}[X|A]$ is $\frac{(i)}{(ii)}$.
The above fraction should be in the simplest form, i.e. $\text{g.c.d} \{ (i), (ii) \} = 1$.
4. Let X and Y be described by the joint distribution

	$X = -1$	$X = 0$	$X = 1$
$Y = -1$	$1/15$	$2/15$	$2/15$
$Y = 0$	$2/15$	$1/15$	$2/15$
$Y = 1$	$2/15$	$2/15$	$1/15$

and answer the following questions.

(a) $\mathbb{E}[X|Y = -1]$ is $\frac{(i)}{(ii)}$.

The above fraction should be in the simplest form, i.e. $\text{g.c.d} \{ (i), (ii) \} = 1$.

(b) $\text{Var}[X|Y = -1]$ is $\frac{(i)}{(ii)}$.

The above fraction should be in the simplest form, i.e. $\text{g.c.d} \{ (i), (ii) \} = 1$.

5. A fair die is rolled.

(a) The expected value given that the roll was even is _____.

(b) The variance of the value given the roll was even is $\frac{(i)}{(ii)}$.

The above fraction should be in the simplest form, i.e. $\text{g.c.d} \{ (i), (ii) \} = 1$.

6. Let $Y \sim \text{Uniform}(\{1, 2, \dots, 15\})$ and let X be the number of heads on Y flips of a fair coin. The expected value of X is _____.

7. Let X be a discrete random variable. Using Chebyshev's inequality the upper bound on the likelihood that X will be more than two standard deviations from its expected value is $\frac{(i)}{(ii)}$.

The above fraction should be in the simplest form, i.e. $\text{g.c.d} \{ (i), (ii) \} = 1$.

8. Let $X \sim \text{Uniform}(\{-2, -1, 0, 1, 2\})$, and let $f(x) = x^2$. Then $\mathbb{E}[f(X)]$ is _____.