For numerical answers with decimal digits please read instructions.

1. A pair of fair dice are thrown. Let X represent the larger of the two values on the dice and let Y represent the smaller of the two values.
   (a) The domain of X and Y, say S, is given by
      i. $S = \{(i,j) : i,j \in \mathbb{N} \text{ and } 1 \leq i,j \leq 6\}$.
      ii. $S = \{i : i \in \mathbb{N} \text{ and } 1 \leq i \leq 6\}$.
      iii. $S = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$.
   (b) The number of elements in S is _____
   (c) The Range of X, R_X, is given by
      i. $R_X = \{1,2,3,4,5,6\}$.
      ii. $R_X = \{6\}$.
      iii. $R_X = \{4,5,6\}$.
   (d) The Range of Y, R_Y, is given by
      i. $R_Y = \{1,2,3,4,5,6\}$.
      ii. $R_Y = \{1\}$.
      iii. $R_Y = \{1,2,3\}$.
   (e) Do X and Y have the same range?
   (f) The probability mass function of X is given by
      i. $f_X(i) = \frac{i}{3}$ for $i \in R_X$
      ii. $f_X(i) = \frac{2i-1}{36}$ for $i \in R_X$
      iii. $f_X(i) = 1$ for $i \in R_X$
   (g) The probability mass function of Y is given by
      i. $f_Y(i) = \frac{2(7-i)-1}{36}$ for $i \in R_Y$
      ii. $f_Y(i) = \frac{i}{3}$ for $i \in R_Y$
      iii. $f_Y(i) = 1$ for $i \in R_Y$
   (h) Is it true that X and Y have the same distribution ?

2. Two dice are rolled. Let X denote the product of the dice and let Y denote the value of the first die.
   (a) $\mathbb{P}(X = 12)$ is $\boxed{\frac{(i)}{(ii)}}$.

   The above fraction should be in the simplest form, i.e. g.c.d $\{\ (i),(ii)\ \}$ =1.

(b) $\mathbb{P}(Y = 4)$ is $\boxed{\dfrac{(i)}{(ii)}}$.

The above fraction should be in the simplest form, i.e. g.c.d $\{\ (i), (ii)\ \}$ *=1.*

(c) $\mathbb{P}(X = 12, Y = 4)$ is $\boxed{\dfrac{(i)}{(ii)}}$.

The above fraction should be in the simplest form, i.e. g.c.d $\{\ (i), (ii)\ \}$ *=1.*

(d) $\mathbb{P}(X = 5)$ is $\boxed{\dfrac{(i)}{(ii)}}$.

The above fraction should be in the simplest form, i.e. g.c.d $\{\ (i), (ii)\ \}$ *=1.*

(e) Is it true that X and Y are independent?

3. Let X and Y be random variables with joint distribution given by the chart below.

	$X = -1$	$X = 0$	$X = 2$
$Y = 0$	1/12	0	3/12
$Y = 1$	2/12	1/12	0
$Y = 2$	3/12	1/12	1/12

Then

(a) $\mathbb{P}(X = 0) =$ is $\boxed{\dfrac{(i)}{(ii)}}$.

The above fraction should be in the simplest form, i.e. g.c.d $\{\ (i), (ii)\ \}$ *=1.*

(b) $\mathbb{P}(Y = 1) =$ is $\boxed{\dfrac{(i)}{(ii)}}$.

The above fraction should be in the simplest form, i.e. g.c.d $\{\ (i), (ii)\ \}$ *=1.*

(c) $\mathbb{P}(X = 2|Y = 2)$ is $\boxed{\dfrac{(i)}{(ii)}}$.

The above fraction should be in the simplest form, i.e. g.c.d $\{\ (i), (ii)\ \}$ *=1.*

(d) $\mathbb{P}(Y = 2|X = 2)$ is $\boxed{\dfrac{(i)}{(ii)}}$.

The above fraction should be in the simplest form, i.e. g.c.d $\{\ (i), (ii)\ \}$ *=1.*

(e) Is it true that X and Y are independent?

4. Suppose S is a sample space consisting of sequences of two coin flips. Let X be a random variable that is 1 if the first coin is heads, and 0 otherwise and Y be a

random variable that is 1 if the first coin is tails, and 0 otherwise, and Z be a random variable that is 1 if the second coin is tails, and 0 otherwise.

(a) Do X, Y, and Z have the same p.m.f.?
   i. Yes
   ii. No

(b) Do the pairs (X, Y) and (X, Z) have same joint p.m.f.?
   i. Yes
   ii. No

(c) Are X and Y independent?
   i. Yes
   ii. No

(d) Are X and Z independent?
   i. Yes
   ii. No

5. An urn has four balls labeled $1, 2, 3$, and 4. A first ball is drawn and its number is denoted by X. A second ball is then drawn from the three remaining balls in the urn and its number is denoted by Y.

(a) $\mathbb{P}(X = 1)$ is $\boxed{\dfrac{(i)}{(ii)}}$.

The above fraction should be in the simplest form, i.e. g.c.d { $(i), (ii)$ } =1.

(b) $\mathbb{P}(Y = 2|X = 1)$ is $\boxed{\dfrac{(i)}{(ii)}}$.

The above fraction should be in the simplest form, i.e. g.c.d { $(i), (ii)$ } =1.

(c) $\mathbb{P}(Y = 2)$ is $\boxed{\dfrac{(i)}{(ii)}}$.

The above fraction should be in the simplest form, i.e. g.c.d { $(i), (ii)$ } =1.

(d) $\mathbb{P}(X = 1, Y = 2)$ is $\boxed{\dfrac{(i)}{(ii)}}$.

The above fraction should be in the simplest form, i.e. g.c.d { $(i), (ii)$ } =1.

(e) Is is true that X and Y are independent?

6. Two dice are rolled. Let X denote the sum of the dice and let Y denote the value of the first die.

(a) $\mathbb{P}(X = 7)$ is $\boxed{\dfrac{(i)}{(ii)}}$.

The above fraction should be in the simplest form, i.e. g.c.d { $(i), (ii)$ } =1.

(b) $\mathbb{P}(Y = 4)$ is $\boxed{\frac{(i)}{(ii)}}$.

The above fraction should be in the simplest form, i.e. g.c.d $\{ (i), (ii) \}$ *=1.*

(c) $\mathbb{P}(X = 7, Y = 4)$ is $\boxed{\frac{(i)}{(ii)}}$.

The above fraction should be in the simplest form, i.e. g.c.d $\{ (i), (ii) \}$ *=1.*

(d) $\mathbb{P}(X = 5)$ is $\boxed{\frac{(i)}{(ii)}}$.

The above fraction should be in the simplest form, i.e. g.c.d $\{ (i), (ii) \}$ *=1.*

(e) $\mathbb{P}(X = 5, Y = 4)$ is $\boxed{\frac{(i)}{(ii)}}$.

The above fraction should be in the simplest form, i.e. g.c.d $\{ (i), (ii) \}$ *=1.*

(f) Is it true that X and Y are independent?

7. Suppose that the number of earthquakes that occur in a year, anywhere in the world, is a Poisson random variable X with mean 5. Suppose that the probability that any given earthquake has magnitude at least 5 on the Richter scale is 1/3. Let Y be the number of earthquakes in year that have magnitude at least 5.

   (a) Using the function `dpois` in `R`, calculate $\mathbb{P}(X = 10)$, the probability that there are 10 earthquakes in a particular year[1]. Choose the correct answer from below.

      i. [1] 0.0241
      ii. [1] 0.0181
      iii. [1] 0.0321

   (b) Using the function `dbinom` in `R`, calculate $\mathbb{P}(Y = 5 | X = 10)$, the probability that there are 5 earthquakes with magnitude at least 5, given that there are 10 earthquakes in a year[2]. Choose the correct answer from below.

      i. [1] 0.1365
      ii. [1] 0.1259
      iii. [1] 0.1482

   (c) Using the function `dpois` in `R`, calculate $\mathbb{P}(Y = 5)$, the probability that there are 5 earthquakes of magnitude at least 5 in a particular year[3]. Choose the correct answer from below.

      i. [1] 0.02146
      ii. [1] 0.02024
      iii. [1] 0.02298

[1]Recall from previous HW: $X \sim Poisson(5)$
[2]Recall from previous HW:: $Y|(X = k) \sim Binomial(k, 1/3)$
[3]Recall from previous HW: $Y \sim Poisson(5/3)$