For numerical answers with decimal digits please read instructions.

1. Banta and Meera bet Rupees 10 each on successive tosses of a fair coin. Each has Rupees 60 in hand and plan to keep betting until one of them runs out of money.

   (a) Probability that they break even after six tosses is $\frac{(i)}{(ii)}$.
   The above fraction should be in the simplest form, i.e. g.c.d $\{ (i), (ii) \} = 1$.

   (b) Probability that Meera wins all the money on the tenth toss is $\frac{(i)}{(ii)}$.
   The above fraction should be in the simplest form, i.e. g.c.d $\{ (i), (ii) \} = 1$.

2. Suppose that airplane engines operate independently in flight and fail with probability p $(0 < p < 1)$. A plane makes a safe flight if at least half of its engines are running. Sobha Airlines has a four–engine plane and Ahmed Airlines has a two–engine plane for a flight from Bangalore to Delhi. Which airline has the higher probability for a successful flight:

   (a) Sobha Airlines is safer when p is less than $\frac{(i)}{(ii)}$.
   The above fraction should be in the simplest form, i.e. g.c.d $\{ (i), (ii) \} = 1$.

   (b) Ahmed Airlines is safer when $1 - p$ is less than $\frac{(i)}{(ii)}$.
   Give your answer as a fraction in the simplest form.

   (c) Both Air lines are equally safe when $p(1 - p)$ is equal to $\frac{(i)}{(ii)}$.
   The above fraction should be in the simplest form, i.e. g.c.d $\{ (i), (ii) \} = 1$.

3. Using the `help()` command understand that the inbuilt R-function `dgeom(x,prob)` returns the probability of getting exactly `x` many tails before a head occurs for the first time in a sequence of coin tossing experiments where `prob` is the probability of a head occurring in each individual trial[1]. Please indicate the correct answer after evaluating the following expressions in `R`:

   (a) `> dgeom(3,0.5)`

      i. [1] 0.0625
      ii. [1] 0.1250
      iii. [1] 0.0025

   (b) `> dgeom(10,0.2)`

[1] This is the setting of the geometric distribution and `dgeom` gives us the probability mass function.

i. [1] 0.02147
ii. [1] 0.03278
iii. [1] 0.03826

4. Using the `help()` command understand that the inbuilt R-function `qbinom(p,size,prob)` returns the smallest number of heads `x` such that the probability of getting fewer than or equal to `x` many heads in `size` many coin tossing experiments is at least `p`, where `prob` is the probability of a head occurring in each individual trial[2]. Please indicate the correct answer after evaluating the following expressions in `R`:

   (a) `> qbinom(0.25,10,0.5)`

   i. [1] 2
   ii. [1] 3
   iii. [1] 4

   (b) `> qbinom(0.9,10,0.75)`

   i. [1] 10
   ii. [1] 9
   iii. [1] 8

   Note: We can use `d`, `p`, `q` along with any distribution name to get the density function, cumulative distribution function and quantile function respectively.

5. Suppose we are given a biased coin with probability of heads being $\frac{1}{3}$. Then

   (a) the probability that a head appears for the first time in the 5-th toss is equal to $\frac{(i)}{(ii)}$.
   The above fraction should be in the simplest form, i.e. g.c.d $\{ (i), (ii) \} = 1$.

   (b) the probability that a tail appears for the fourth time in the 5-th toss is equal to $\frac{(i)}{(ii)}$.
   The above fraction should be in the simplest form, i.e. g.c.d $\{ (i), (ii) \} = 1$.

6. There are 150 students in the Probability 101 class. Of them, ninety are female, sixty use a pencil (instead of a pen), and thirty are wearing eye glasses. A student is chosen at random from the class. Define the following events:

$$A = \{\text{the student is a female}\}$$

$$B = \{\text{the student uses a pencil}\}$$

$$C = \{\text{the student is wearing eye glasses}\}$$

   Is it possible for these events to be mutually independent ?

[2] This is the setting of the binomial distribution and `qbinom` gives us the quantile function.