For numerical answers with decimal digits please read instructions.

1. Flip a fair coin four times and record the sequence of heads and tails. Let S be the sample space of all sixteen possible orderings of the results.

   (a) Then $\mathbb{P}$(Observing two tails in four flips) is equal to $\frac{(i)}{(ii)}$.
   The above fraction should be in the simplest form, i.e. g.c.d $\{\ (i), (ii)\ \}$ *=1.*

   (b) $\mathbb{P}$(First flip is tail) is equal to $\frac{(i)}{(ii)}$.
   The above fraction should be in the simplest form, i.e. g.c.d $\{\ (i), (ii)\ \}$ *=1.*

2. Suppose there are only thirteen teams with a non-zero chance of winning the next World Cup, namely Spain (with a 14% chance), the Netherlands (with a 11% chance), Germany (with a 11% chance), Italy (with a 10% chance), Brazil (with a 10% chance), England (with a 9% chance), Argentina (with a 9% chance), Russia (with a 7% chance), France (with an 6% chance), Turkey (with a 4% chance), Paraguay (with a 4% chance), Croatia (with a 4% chance) and Portugal (with a 1% chance).

   (a) The probability that the next World Cup will be won by a South American country is equal to $\frac{(i)}{(ii)}$.
   The above fraction should be in the simplest form, i.e. g.c.d $\{\ (i), (ii)\ \}$ *=1.*

   (b) The probability that the next World Cup will be won by a country that is not from South America is equal to $\frac{(i)}{(ii)}$.
   The above fraction should be in the simplest form, i.e. g.c.d $\{\ (i), (ii)\ \}$ *=1.*

3. Nitya, a biologist is modeling the size of a frog population in a series of ponds. She is concerned with both the number of egg masses laid by the frogs during breeding season and the annual precipitation into the ponds. She knows that in a given year there is an 86% chance that there will be over 150 egg masses deposited by the frogs (event E) and that there is a 64% chance that the annual precipitation will be over 17 inches (event F).

   (a) In terms of E and F, the event "there will be over 150 egg masses and an annual precipitation of less than or equal to 17 inches" is :

      i. $E^c \cap F$
      ii. $E \cap F$
      iii. $E \cap F^c$
      iv. $E^c \cap F^c$

   (b) In terms of E and F, the event "there will be 150 or fewer egg masses or the annual precipitation will be over 17 inches" is :

      i. $E^c \cup F$

ii. $E \cup F$

iii. $E^c \cup F^c$

iv. $E \cup F^c$

(c) Suppose the probability of the event from 3(a) is 0.59. The probability of the event from 3(b) is equal to $\frac{(i)}{(ii)}$.

The above fraction should be in the simplest form, i.e. g.c.d $\{ (i), (ii) \}$ *=1.*

4. Suppose we roll a fair die and therefore the set of all outcomes is given by $S = \{1, 2, 3, 4, 5, 6\}$.

(a) The probability of each individual outcome is equal to $\frac{(i)}{(ii)}$.

The above fraction should be in the simplest form, i.e. g.c.d $\{ (i), (ii) \}$ *=1.*

(b) The probability of the event $E = \{1, 3, 4, 6\}$ is equal to $\frac{(i)}{(ii)}$.

The above fraction should be in the simplest form, i.e. g.c.d $\{ (i), (ii) \}$ *=1.*

5. Suppose E and F are events in a sample space S. Suppose that $\mathbb{P}(E) = 0.7$ and $\mathbb{P}(F) = 0.5$.

(a) The largest possible value of $\mathbb{P}(E \cap F)$ is equal to $\frac{(i)}{(ii)}$.

The above fraction should be in the simplest form, i.e. g.c.d $\{ (i), (ii) \}$ *=1.*

(b)The smallest possible value of $\mathbb{P}(E \cap F)$ is equal to $\frac{(i)}{(ii)}$.

The above fraction should be in the simplest form, i.e. g.c.d $\{ (i), (ii) \}$ *=1.*

6. Akshaya has lost her room key. The hostel warden gives her 50 keys and tells her that one of them will open her room. She decides to try each key successively and notes down the number of the attempt at which the room opens.

(a) Sample space for this experiment is:

i. $\{1, 2, 3, \ldots, 50\}$

ii. $\{1, 2, 3, \ldots, 49\}$

iii. $\{2, 3, 4, \ldots, 50\}$

iv. $\{2, 3, 4, \ldots, 49\}$

(b) Probability that Akshaya opens her room in the n-th attempt is:

i. $\frac{1}{50}\left(\frac{49}{50}\right)^{n-1}$

ii. $\frac{1}{50}$

iii. $\binom{50}{n}\left(\frac{1}{2}\right)^{50}$

iv. $\frac{1}{n}$

7. Please calculate using `R`-software using `fractions(dbinom(..))`

(a) We flip a fair coin 120 times and record the sequence of heads and tails. `R`-software to calculate $\mathbb{P}$(Observing 64 tails in 120 flips) and express it as a fraction in the simplest form.

i. [1] 7489/374762
ii. [1] 5660/101463
iii. [1] 1432/873659

(b) We have a *biased* coin such that $\mathbb{P}(\text{head}) = 0.25$. We flip it 48 times and record the sequence of heads and tails. Use `R`-software to calculate $\mathbb{P}(\text{Observing 12 heads in 48 flips})$ and express it as a fraction in the simplest form.

i. [1] 27231/206320
ii. [1] 46583/928374
iii. [1] 87927/177854