For numerical answers with decimal digits please read instructions.

1. Let $X \sim$ Normal $(0, 1)$.
   (a) Let $r > 0$ and define $Y = rX$. Then
      i. $Y \sim$ Normal$(0, r^2)$.
      ii. $Y \sim$ Normal $(0, r)$.
      iii. $Y \sim$ Normal$(0, \frac{r}{3})$.
      iv. None of the above
   (b) Let $Y = -X$. Then
      i. $Y \sim$ Normal $(-1, 1)$.
      ii. $Y \sim$ Normal$(0, 1)$.
      iii. $Y \sim$ Normal$(0, 0)$.
      iv. None of the above
   (c) Let a and b be real numbers with $a < b$ and let $Y = (b - a)X + a$. Then
      i. $Y \sim$ Normal$(a, b - a)$
      ii. $Y \sim$ Normal$(0, b - a)$
      iii. $Y \sim$ Normal$(a, (b - a)^2)$
      iv. None of the above

2. Decide which of the following $f : \mathbb{R} \to \mathbb{R}$ can be a probability density function.
   (a) $f(0) = 1$, $f(x) = 2$ for all $x \in (0, \frac{1}{2})$, and $f(x) = 0$ otherwise.
   (b) $f(x) = 7$ for all $x \in (0, \frac{1}{2})$, and 0 otherwise.
   (c) $f(\frac{3}{4}) = -8$, $f(x) = 2$ for all $x \in (0, \frac{1}{2})$, and 0 otherwise.

3. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} C \cdot \sin(x) & \text{if } 0 < x < \pi, \\ 0 & \text{otherwise.} \end{cases}$$

   (a) The value of C that makes $f(x)$ a probability density function is $\frac{(i)}{(ii)}$.
   The above fraction should be in the simplest form, i.e. g.c.d { (i), (ii) } =1.
   (b) Let $\mathbb{P}$ be the probability whose density is f then $\mathbb{P}((\frac{1}{2}, 1))$ is ____
   Gaive your answer rounded up to four decimal digits.
   (c) Is it true $\mathbb{P}((0, \frac{1}{4})) < \mathbb{P}((\frac{1}{4}, \frac{1}{2}))$?

4. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} k & \text{if } 0 < x < \frac{1}{4}, \\ 2k & \text{if } \frac{1}{4} \leq x < \frac{3}{4}, \\ 3k & \text{if } \frac{3}{4} \leq x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Then the value of k that makes f a probability density function is $\frac{(i)}{(ii)}$.

The above fraction should be in the simplest form, i.e. g.c.d $\{\ (i), (ii)\ \}$ *=1.*

5. Let $k > 0$ and $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} ke^{-x} & \text{if } x > 0, \\ 0 & \text{otherwise.} \end{cases}$$

be a probability density function. Let $\mathbb{P}$ be the probability whose density is f then $\mathbb{P}((0, \ln(2)))$ is $\frac{(i)}{(ii)}$.

The above fraction should be in the simplest form, i.e. g.c.d $\{\ (i), (ii)\ \}$ *=1.*

6. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} \frac{2^b}{c} x^4 \exp(-2x) & \text{if } 0 < x, \\ 0 & \text{otherwise.} \end{cases}$$

Suppose f is a probability density function. Then the value of b and c are given by

(a) $b = 5$ and $c = 24$
(b) $b = 6$ and $c = 10$
(c) $b = 4$ and $c = 24$
(d) $b = 5$ and $c = 6$

7. Suppose X is a random variable with density

$$f(x) = \begin{cases} cx^2(1-x) & \text{for } 0 \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Find:

(a) The value of c is ______.

(b) The conditional probability $\mathbb{P}(X > 0.2 \mid X < 0.5)$ is $\frac{(i)}{(ii)}$.

The above fraction should be in the simplest form, i.e. g.c.d $\{\ (i), (ii)\ \}$ *=1.*

8. The “median” of a continuous random variable X is a value of x for which

$$P(X > x) = P(X < x) = \frac{1}{2}.$$

(a) If $X \sim \text{Uniform}(11, 15)$ then the median of X is _____.

(b) If $Y \sim \text{Exp}(\ln 2)$ then the median of Y is _____.

(c) If $Z \sim \text{Normal}(5, 7)$ then the median of Z is _____.

9. Let $X \sim \text{Exp}(1)$. The “α-th percentile” is a value b such that $\mathbb{P}(X < b) = \frac{\alpha}{100}$.

i. The 25-th percentile of X is equal to _____.
Give your answer rounded up to four decimal digits

ii. The 50-th percentile of X is equal to _____.
Give your answer rounded up to four decimal digits

iii. The 75-th percentile of X is equal to _____.
Give your answer rounded up to four decimal digits