For numerical answers with decimal digits please read instructions.

1. Let $X \sim \text{Uniform}(\{1,2,3\})$ and $Y \sim \text{Uniform}(\{1,2,3\})$ be independent and let $Z = X + Y$.

   (a) The range of Z is:

      i. 2,3,4,5
      ii. 2,3,4,6
      iii. 2,3,4,5,6
      iv. None of the above

   (b) $\mathbb{P}(Z = 3) = \frac{(i)}{(ii)}$.
   The above fraction should be in the simplest form, i.e. g.c.d $\{\ (i), (ii)\ \}$ *=1.*

   (c) $\mathbb{P}(Z = 5) = \frac{(i)}{(ii)}$.
   The above fraction should be in the simplest form, i.e. g.c.d $\{\ (i), (ii)\ \}$ *=1.*

   (d) Is Z uniformly distributed over its range?

      i. Yes
      ii. No

2. Let X, Y be two random variables with joint probability mass function given by

$$\mathbb{P}(X = i, Y = j) = \frac{1}{2^{i+j}}, \qquad i, j \in \mathbb{N}.$$

   Then

   (a) $\mathbb{P}(X + Y = 4)$ is $\frac{(i)}{(ii)}$.
   The above fraction should be in the simplest form, i.e. g.c.d $\{\ (i), (ii)\ \}$ *=1.*

   (b) $\mathbb{P}(X - Y = 2)$ is $\frac{(i)}{(ii)}$.
   The above fraction should be in the simplest form, i.e. g.c.d $\{\ (i), (ii)\ \}$ *=1.*

3. Three dices are rolled. Probability that the sum of three dice will equal six is $\frac{(i)}{(ii)}$.
   The above fraction should be in the simplest form, i.e. g.c.d $\{\ (i), (ii)\ \}$ *=1.*

4. Let X and Y be independent random variables each geometrically distributed with parameter p.

   (a) Given $p = 0.5$, $\mathbb{P}(\min(X, Y) = 5)$ is $\frac{(i)}{(ii)}$.
   The above fraction should be in the simplest form, i.e. g.c.d $\{\ (i), (ii)\ \}$ *=1.*

(b) Given $p = \frac{1}{3}$, $\mathbb{P}(\min(X,Y) = X)$ is $\dfrac{(i)}{(ii)}$.

The above fraction should be in the simplest form, i.e. g.c.d $\{\ (i), (ii)\ \}$ *=1.*

(c) Given $p = \frac{1}{8}$, then $\mathbb{P}(X + Y = 5)$ is $\dfrac{(i)}{(ii)}$.

The above fraction should be in the simplest form, i.e. g.c.d $\{\ (i), (ii)\ \}$ *=1.*

(d) Given $p = 0.6$, $\mathbb{P}(X > 11 | X > 5)$ is $\dfrac{(i)}{(ii)}$.

The above fraction should be in the simplest form, i.e. g.c.d $\{\ (i), (ii)\ \}$ *=1.*

(e) Let $0 < p < 1$, $\mathbb{P}(Y = 5 | X + Y = 15)$ is $\dfrac{(i)}{(ii)}$.

The above fraction should be in the simplest form, i.e. g.c.d $\{\ (i), (ii)\ \}$ *=1.*

5. Let $0 < p, q < 1$, $X \sim \text{Bernoulli}(p)$, and $Y \sim \text{Bernoulli}(q)$ be independent.

   (a) Is XY a Bernoulli random variable?

      i. Yes

      ii. No

   (b) Is $X + Y - XY$ a Bernoulli random variable?

      i. Yes

      ii. No