Basic Concepts in Probability :-

Question :- What is the likelihood of rain tomorrow?

Ans :- The chance of rain tomorrow is 10%.

Probability / statistics : determine how likely certain things are going to occur.

Framework : What are all the possibilities that are going to occur?

Definition 1.1.1 [Sample Space] A sample space S is a set. The elements of S will be called "outcomes" and S should be viewed as a listing of all possible outcomes that might occur. We will call the process of actually selecting one of these outcomes an "experiment".

Example :- $S = \{1, 2, 3, 4, 5, 6\}$ – Rolling a die and noting the number on the top face

$S = \{\text{Heads}, \text{Tails}\}$ – tossing a coin and noting down the label on the top face.

Definition 1.1.2. [Event - Temporary Definition] Given a sample space S, an "event" is any subset $E \subseteq S$.

- allows us to talk about how certain events / a particular range of possible outcomes might occur.

Example (Contd.): $E = \{1, 3, 5\}$ = an odd number occurs when we roll a die

- S - is also an event, ϕ is an event

Next:- To each event $E \subseteq S$ we want to assign a chance (or Probability) which will be a number between 0 and 1.

Example (Contd.): "Probability" of Heads in toss of a fair coin is $1/2$. $E = \{Heads\}$ & assign: $\mathbb{P}(E) = \frac{1}{2}$

Definition [Axioms of Probability - Kolmogorov]

Let S be a sample space and $\mathcal{F}$ be the collection of all events. A "Probability" is a function $\mathbb{P} : \mathcal{F} \longrightarrow [0,1]$ such that

1. $\mathbb{P}(S) = 1$; and
2. $E_1, E_2, \ldots$ be a countable collection of disjoint events (ie $E_i \cap E_j = \phi$, $i \neq j$) then

$$\mathbb{P}\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{j=1}^{\infty} \mathbb{P}(E_j) \quad - (*)$$

Remarks:-

Axiom 1: S — list of all possible outcomes

Axiom 2 — not that complicated — Probabilities of a union of disjoint events is the sum of probabilities.

- in (*), series of R.H.S converges
- Countable additivity is important.